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THE PHYSICAL REVIEW.

ANNOUNCEMENT OF THE TRANSFER OF THE REVIEW TO THE AMERICAN PHYSICAL SOCIETY.[1]

WITH the present number the AMERICAN PHYSICAL SOCIETY takes over the PHYSICAL REVIEW and assumes the entire responsibility for its conduct. In so doing the society wishes to give expression to its deep appreciation of the great service done to physics and physicists in America by the editors who, in July-August, 1893, put forth the first number of a new journal, and to Cornell University, which assumed the financial risk. There was at that time no journal in this country entirely devoted to physics, and there was no national society. During nearly twenty years the original editors have carried on the arduous task of maintaining this journal on a high standard, and it is difficult to estimate the value of their efforts in furthering the cause of physics in America. In this manner the way for the foundation of the American Physical Society was prepared, and early in its history the society and the REVIEW entered into relations which have continually become closer. The former editors have now thought best to complete their task by transferring their control to the American Physical Society, and the PHYSICAL REVIEW now becomes the journal of that society, national in scope, and looks for the coöperation of all American physicists. A. G. W.

[1] For a more detailed statement of the action of the Society in taking over the REVIEW and of the arrangements made for its future management, see the Minutes of the Sixty-fifth meeting, pages 61–66, in this number.

DIFFRACTION AND SECONDARY RADIATION WITH ELECTRIC WAVES OF SHORT WAVE-LENGTH.

BY A. D. COLE.

SEVERAL years ago the writer presented to the American Physical Society the results of a study of diffraction effects obtained with electric waves of short wave-length.[1] In that work, as has been the usual practice among experimenters with electromagnetic radiation of wave-length less than a meter, the radiating source was placed at the focus of a parabolic cylindrical mirror. With this arrangement the experiments find their optical analogy in diffraction phenomena of the type commonly called the "Fraunhofer class," where the light used has a plane wave-front. In the experiments about to be described, however, the exciter of electric waves was provided with no converging arrangement, either mirror or lens. Thus a "point source" (or more strictly a line source) was approximated, so that the results have their optical analogue in the "Fresnel class" of optical experiments. This present study differs from the earlier one also in this point; that the receiver also was unprovided with any arrangement for intensifying the effect of the radiation upon it. The usual parabolic mirror would be inadmissible in any study which attempts to compare intensities for points only a few centimeters apart, but in the earlier work a narrow reflecting strip about 1.5 cm. wide was placed behind the exciter at about one quarter wave-length distance, as suggested by Righi.[2] In the present experiments however, as it was desired to keep the conditions as simple as possible, both receiver and exciter were used alone.

The exciter used was of the modified Righi type described in earlier papers.[3] The oscillating system consisted of two cylinders of aluminium, each 2.4 cm. long and .32 cm. in diameter. They were placed with axis vertical and separated by a minute spark gap, whose length could be adjusted by a slow-motion screw pressing upon one limb of the glass frame, which served as a support for the two cylinders and the oil reservoir surrounding the spark gap. The two auxiliary spark gaps in air were from 2 to 3 mm. long. The receiver was of the Klemencic thermal type.

[1] Science, April 5, 1907 (Report of Meeting of Section B, A. A. A. S.).

[2] Righi, Die Optik der elektrischen Schwingungen, p. 36 (1898).

[3] PHYS. REV., 33, p. 241, 1906.

A thermal receiver was preferred because its indications are proportional to the energy received and it was desired to study the distribution of the energy and not simply locate maxima and minima. The receiver consisted of two brass cylinders, having in place of the spark gap a thermo-junction of fine iron and constantin wire of .02 mm. diameter. These cylinders were smaller and shorter than those of the exciter, but could be lengthened by fitting over them a pair of larger cylinders. These larger cylinders were of the same diameter at those of the exciter and had a hole bored axially in one end, of diameter equal to that of the small cylinders (.21 cm.). This end was also provided with a diametral slit to give springiness and range of adjustment. Special care was taken to insure a close fit and firm pressure of the sliding cylinders upon the smaller fixed cylinders. To adapt the same receiver for use with exciters of different period and to be able to tune to resonance conditions in each case, eleven pairs of sliding cylinders were prepared, varying in length from 5 to 44 mm.[1] In the present work however only the one exciter described was used; its length was 4.8 cm. and the receiver was found by a series of careful experiments to be in resonance with it—as will be shown presently—when its length was 3.9 cm.

DIFFRACTION EXPERIMENTS.

Exciter and receiver having been brought to the same vibration period by adjustment of the length of the receiver, some experiments were tried with such conditions as would produce diffraction if light radiation were used. Naturally it was found impracticable to imitate optical experiments in the relation existing between wave-length and distances travelled. Satisfactory intensity was obtained only when the total distance of travel was less than a meter, *i. e.*, only a few wave-lengths. Even at such distances the required intensity is not easy to get when no mirror or lens is used with either exciter or receiver unless one uses an interrupter of high frequency. In the present case a Wehnelt interrupter was used, in connection with an induction coil capable of giving a 12 cm. spark.

Fig. 1 shows the result of an experiment to measure the diffraction due to a thin edge of conducting material. Exciter and receiver were placed 60 cm. apart, with the axis of each vertical. Halfway between them was a flat sheet of zinc 47 cm. high by 37 wide, the center of one vertical edge being on the line *ER*. (The metal screens are not shown full size in the figures.) The receiver was mounted at *R* on a revolving arm which could swing horizontally about a center immediately below the refracting edge. The arm was swung about, step by step, through an angle of

[1] PHYS. REV., 20, p. 269, 1905.

55 degrees, from 15 degrees behind the screen edge to 40 degrees on the other side of the axial line. Readings were taken at each 5°, four in each position. Five times at approximately equal intervals during the readings the intensity was read with screen removed and receiver in the central position. There was a constant and gradual diminution of intensity during the series due to the gradual lengthening of the spark gap. The rate of this diminution is shown by the slope of the dotted line in the figure. To compare the readings for the several positions more fairly, the mean for each position was corrected to the value corresponding to the mean intensity for the series. From the figure we observe that along the line of the edge the intensity is reduced to one fifth that obtained when the screen is removed. It falls off gradually behind the edge, reaching value one tenth at about 8° and thereafter changing but little. On the other side of the axis it rises, more rapidly than it fell away on the protected side, and with increasing rapidity until at 27° from the center the same intensity is reached as with the screen away. Beyond this point the increase is less rapid, but the effect is everywhere greater than with the plate away (so that the word screen becomes a misnomer). A similar experiment was tried with the distance *SR* increased to 50 cm. (making *ER* 80 cm.), but no significant difference appeared with the made to changed distance. If with the greater distance the receiver had been travel along the tangent at *R* instead of along the arc, it might have been possible to get diffraction bands as in the corresponding optical experiment, for then two half period elements of the wave front would become exposed when *R* was removed about 50 cm. to one side of the axial line and the conditions for a minimum reached.

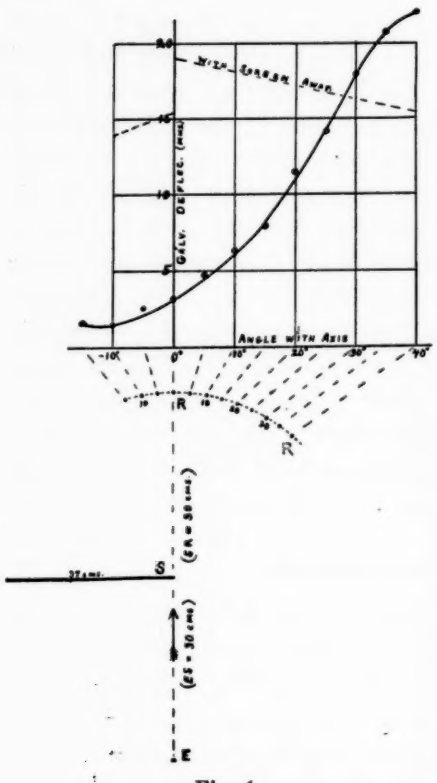


Fig. 1.

In Fig. 2 the results of an experiment are shown where two such plates were used, with their inner edges 20 cm. apart. In this case the radiation

passed through an opening of 20 cm., or about 1.5 wave-lengths in width. The total distance of the receiver from the exciter was 80 cm., and from the plane of the plates was 50 cm. Readings were taken for positions 5° apart through an angle of 50° from the axis. The curve shows the

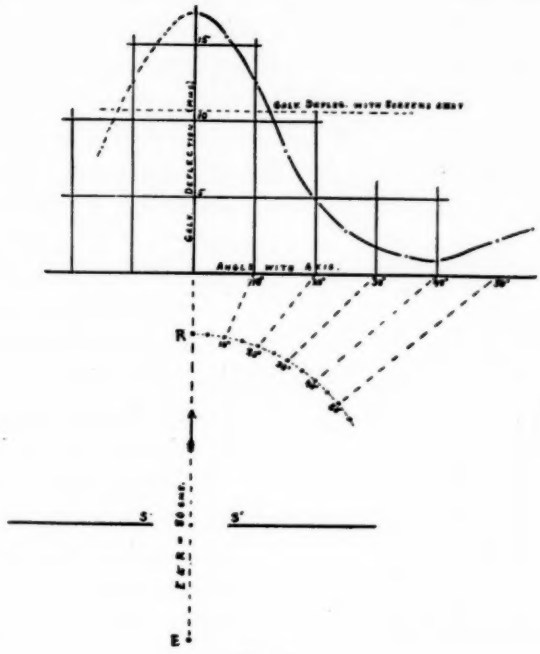
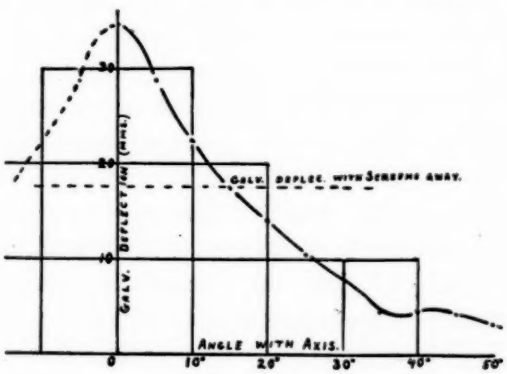


Fig. 2.

energy distribution found. It is seen that the effect of the plates was to increase the energy on the axial line by more than 50 per cent. above that shown when the plates were removed. As R is swung out to one side



OPENING BETWEEN SCREENS 25 CMS.

Fig. 3.

the energy received decreases, slowly at first and then more rapidly. At 12° the same intensity is indicated as with the plates removed (shown on the curve by the horizontal dotted line). At 30° the geometrical shadow of the nearer plate is entered and the intensity is now about one sixth of that amount. At about 40° a minimum appears with intensity about 5 per cent. of that along the axis. (The readings here however were small and somewhat irregular and another comparison of the energies at 40° and 55° showed them nearly equal.)

A similar series was taken with a 25 cm. opening between the plates. The result is shown in Fig. 3. In this case the energy along the axis is twice that found when the plates are removed. The rate of falling off is somewhat more rapid than with the 20 cm. opening.

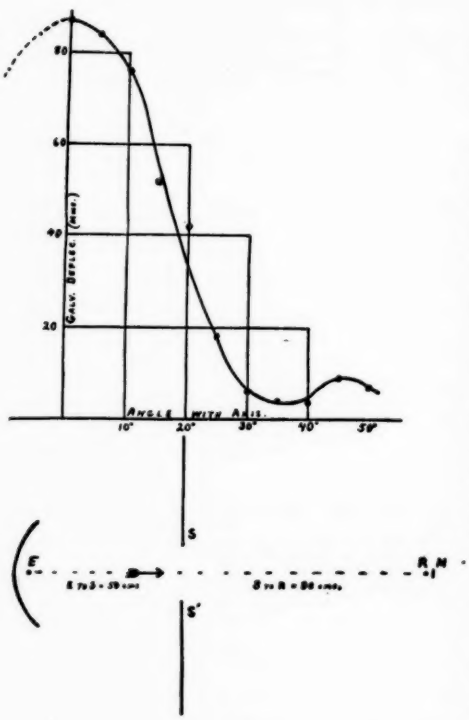


Fig. 4.

A clearer indication of maxima and minima ("fringes") is shown in the experiment which furnished the data for Fig. 4. This however was an experiment of the Fraunhofer class, since the exciter was placed at the focal point of a concave parabolic mirror (of 7.5 cm. focal length and 35 cm. width of aperture). A narrow reflecting strip of sheet metal was placed behind the receiver. Thus larger readings were secured. The distance between plates was also slightly different (17.6 cm. instead of 20). A minimum appears in the curve at about 37° and a second weak maximum shows beyond.

In Fig. 5 is shown the effect when the plates are located with reference to the exciter as shown in Fig. 2, but the receiver is placed successively at seven different points on the axis. Curve *A* shows the change in intensity as the receiver is moved from a position 40 cm. from the exciter by 10 cm. steps until it was 100 cm. away. Curve *B* shows the result of the same changes, when the plates were taken away. In every position of the receiver it is seen that the presence of the plates increases the

amount of energy received. This increase is most marked when the receiver is 50 cm. distant where the ratio of ordinates of the two curves is greater than 2 : 1. The difference is not so great at 60 and 70 cm.

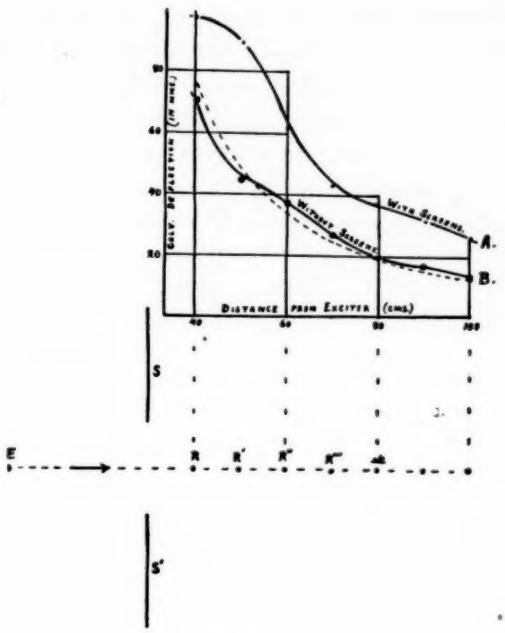


Fig. 5.

distance, but increases again beyond. Curve *B* shows that without the screens the decrease of energy with increase of distance approximately follows the inverse square law. The dotted curve is drawn so that its ordinates are inversely as the squares of the distances between exciter and receiver and this curve approximately coincides with curve *B*.

In another experiment the distance between exciter and receiver was kept fixed at 80 cm. but the plates were shifted along by steps of 20 cm., taking in succession the seven positions indicated in the lower part of Fig. 6. They were in each case 25 cm. apart, with the radiation axis half way between them. As before the effect of the plates is everywhere to increase the energy received at *R*, as shown in the upper part of Fig. 6, where the ordinates show the relative amounts of energy received for the several positions of the screens. The increase is most marked when the plates are 20 cm. from the exciter and is almost zero when the exciter is in the plane of the plates. For other positions the increase lies between 30 and 60 per cent.

Fig. 7. shows the results obtained when the positions of exciter and receiver and the plane of the plates are kept unchanged throughout the series, but the width of the opening between the plates is changed by short steps from 0 to 24 cm. The data for the full-line curve were obtained with the exciter and receiver 60 cm. apart and with the plates half way between them as shown in the lower part of the figure. The data for the dotted curve were obtained with R 20 cm. more distant, *i. e.*, 50 cm. beyond the plane of the plates. The form of the curve is about the same in each case. In both we notice that the radiation received at R is very small when the distance between the plates is less than one fourth the wave-length (3 cm.). Then it rises rapidly and nearly

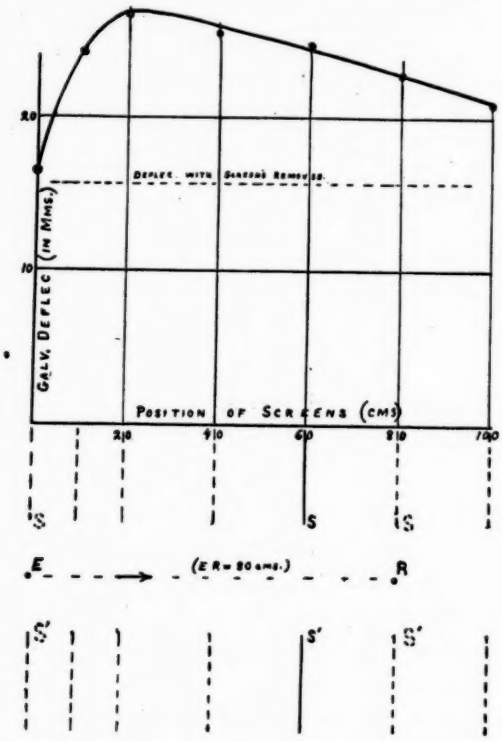


Fig. 6.

in proportion to the width of the opening up to about 1.5 λ (18 cm.). Then the increase is slower up to 24 cm. In the experiment where the total distance is 60 cm., an effect equal to that which we have with the plates removed is secured when the opening is about one wave-length in width. The largest effect is about 60 per cent. greater than this.

A few words about the relative dimensions of exciter and receiver and

the wave-length used in these experiments. Early in the course of this work a paper was published by Dr. J. E. Ives[1] in which a different ratio of length of exciter to receiver was found for resonance than that which the author had named in a paper in PHYS. REV., 20, p. 268. In the

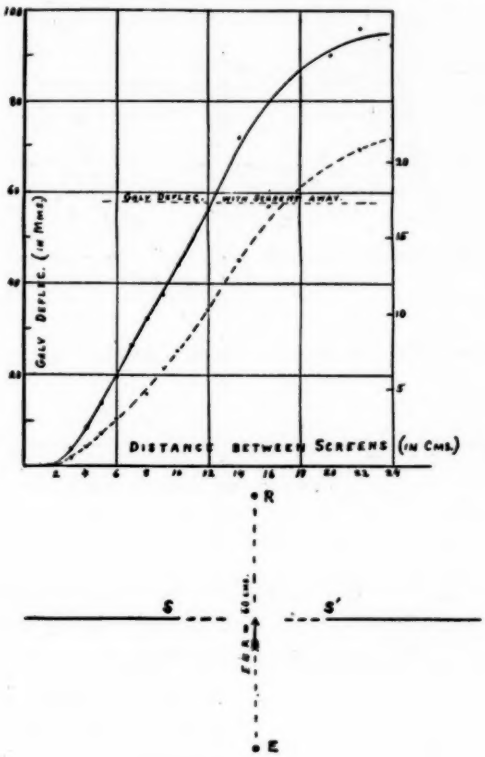


Fig. 7.

earlier experiments the adjustment of length of the receiver was secured by sliding slitted tubes of thin sheet copper over the copper cylinders of the receiver. Dr. Ives's results renewed a suspicion that these thin tubes did not fit as tightly as they ought and perhaps were too light in metal also. Therefore a new receiver was made provided with the set of heavier and better-fitting cylinders described early in this paper. These were adjusted as described in the earlier paper. The results are shown in the six curves of Fig. 8. These all show a decided and rather sharp maximum at about 40 mm.—perhaps the best mean value is 39 mm. The exciter length was 48 mm., and the ratio of lengths is therefore 1.23, which lies between the value previously found and that of Ives, although it is considerably nearer to Ives's value (1.43 for cylinders of 2.5 mm. diam.).

[1] PHYS. REV., 30, pp. 199–221, 1910.

I have not certainly located the cause of error in the early experiment, but it is very likely in the imperfect fit of the sliding cylinders. In the present work the good agreement of the several experiments was obtained

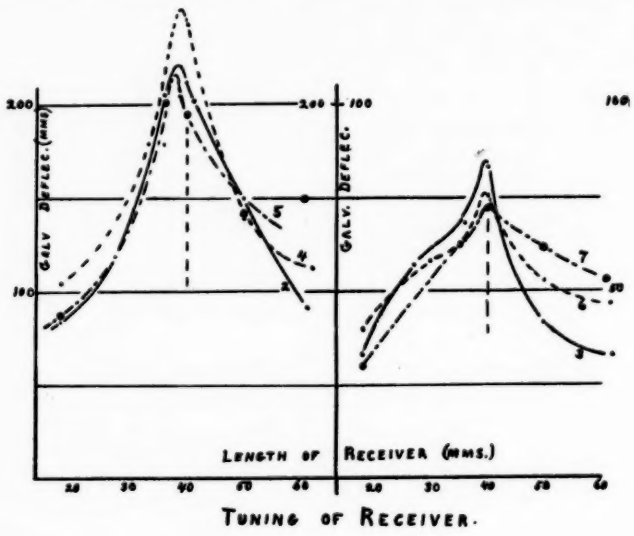


Fig. 8.

in spite of the fact that the conditions were purposely changed in the series shown by the different curves and one other not shown. Thus in one case both exciter and receiver were in concave mirrors, in another neither of them, in the others the exciter alone had a mirror; the distance between exciter and receiver was in one case 30 cm., in another 50 and in others 75; the length of the spark gap was also varied in different experiments.

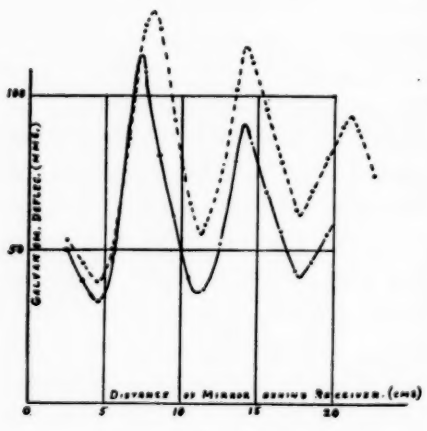


Fig. 9.

Exciter and receiver having been brought into resonance, the wave-length was determined from the interference curve secured by shifting the distance of a large plane sheet of zinc mounted behind the receiver. The data for two such series are shown in Fig. 9. In general each point on one of these curves is the mean of five readings. Thus the dotted curve shown involves over a hundred galvanometer deflections. The

two curves shown were taken more than a week apart. The wave-length figured from the dotted curve is 13.1 cm. The curves seem satisfactory so far as regularity is concerned, also in the ratio of maxima to minima ordinates and in their agreement with each other. Many other curves were taken indicating the same wave-length. This wave-length is however larger than has been found for apparatus of about these dimensions by Ives and others using the interferometer method. If there is anything radically wrong in the method here used it would be desirable to know it, as it is essentially that used by Hertz in his classical interference-by-reflection experiments. It has been shown by the author[1] also that results by this method agree with those given by the Boltzmann two-mirror method and by the method of reflection from a single mirror behind the exciter.

EXPERIMENTS WITH INTERPOSED RODS.

Some measurements were made to find the magnitude of the screening effect of various conductors placed between exciter and receiver. First a thin brass rod was used, 40 cm. long and .32 cm. in diameter. It was placed in a vertical position on the line of exciter and receiver, which were 80 cm. apart. For each position along the line a series of readings was taken as follows: First three or four readings with no rod, then an equal number with the rod in position, again several readings with the rod removed, and so on until ten or twelve readings of each kind were obtained. The screening effect found was surprisingly large for any position, but differed considerably for different distances from the exciter as the following tabulation shows. The first column shows the distance of the rod from the receiver and the second the per cent. of radiation intercepted.

Rod from *R*, cm.	Intercepted by Rod, Per Cent.
2.4	77
5.	60
7.5	44
10.	37
15.	26
20.	27
25.	24
40.	32

One typical set of readings is shown, each of the eight positions tabulated having a similar set.

[1] PHYS. REV., 20, p. 271, 1905, and 23, p. 244, 1906.

Rod Away.		Rod 10 cm. from R.
		14.
		14.2
22.2	←	14.5
22.0		
25.0	→	15.0
		15.1
22.8	←	14.2
22.9		
23.7	→	14.2
		15.6
		14.0
22.0	←	14.4
21.5		
23.0		
25.0		
Means....23.0		14.5

$\frac{14.5}{23}$ = 63 per cent. passing,

$\therefore$ 37 per cent. intercepted.

In the above experiments great care was taken to have the thin rod exactly on the line connecting exciter and receiver. But inasmuch as the diameter of the rod was only about one fortieth of a wave-length, it was difficult to think of so large a screening as a shadow or as a diffraction effect. The effect of moving the rod to one side was therefore tried. It was placed on the central line and half-way between E and R. Then it was moved 2 cm. to one side and a series of readings taken. So also at 4 and 6 cm to one side. In all three positions a large screening effect was found and nearly the same for each, about 25 per cent. Plainly this is no diffraction effect. Similar experiments with a thin wire are later described.

Effects of Lateral Shifting.

The thin rod was now replaced by a narrow strip of sheet metal, 3 cm. wide (and 34 cm. high). Its screening effect was measured on the central line, at the quarter, half and three quarter points (reckoning from R to E). It proved to be 49 per cent., 52 per cent. and 54 per cent. at the three points. The effect of moving this strip also to one side of the central position was tried, and with a surprising result. When 5 cm. from the central position, it cut down the effect at R more than when directly in line. The sidewise motion was continued, a series of readings being taken for each 5 cm. shift. The result is shown graphically in curve a of Fig. 10. At 15 cm. to one side it actually exerted a helpful influence, for the radiation received at R was greater than when the strip was removed.

Next a wider strip was tried in the same way. It was 7.5 cm. wide. The result is shown in curve b of Fig. 10. The effect is evidently of

the same kind but greater. The maximum and minimum occur at the same points but they are more pronounced.

But the greatest surprise came when the last experiment was repeated with the thin edge turned toward the exciter. Almost the same curve as before was obtained, except that in the central position the obstructing power of the piece of metal was distinctly *greater* when it stood edgewise than when its broad side was presented. The results are shown graphically by the points surrounded by small squares in Fig. 10. It is seen that they all lie very close to the curve *b* except the one representing the deflection in the central position, whose height is scarcely two thirds of the ordinate of the curve at that point. The same experiment was then tried with the .32 cm. rod (used in the first test along the axis) The result appears in curve *c* of Fig. 10. It is of the same general character as the others, but the maximum and minimum are less pronounced.

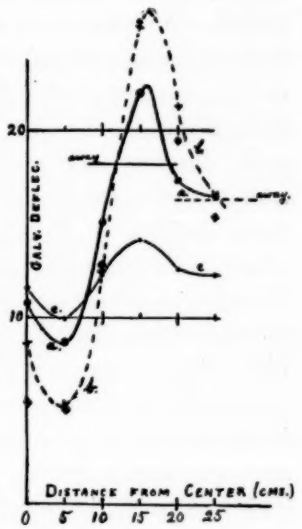


Fig. 10.

These phenomena seem hard to explain from the standpoint of the optical analogy, or if regarded as diffraction effects. A leading physicist suggested that the obstructing effect was proportional to the amount of metal present in the obstruction. This seems unlikely however, as any absorbed energy would doubtless take the form of alternating currents of a frequency so enormous that they would appear as "ultra-skin-currents" (as they have been aptly termed). Experimental evidence that it is not a question of mass of metal was obtained by some measurements on the obstructing effect of a vertical wire having a diameter of .72 mm., much smaller than the thin rod before used. It was found to absorb 19 per cent. of the energy when placed at the half way point and more than 40 per cent. when 5 cm. in front of the receiver. The absorbing power of the 3.2 mm. rod was 32 per cent. and 60 per cent. in the same positions, less than twice as much, while its mass was about twenty times that of the thin wire. That the result is not determined by the mass of metal is also seen by comparing curves *c* and *a*. The characteristic effect is much more marked in curve *a*, due to the thin strip, than it is in curve *c* due to the rod; yet the mass of the strip is smaller than that of an equal length of the rod in the ratio of 14.5 to 23.

The obstructing effect of the thin wire was so great when placed parallel

to the electric force that it was tried in the horizontal position (perpendicular to the electric force), but no absorbing power could then be detected. A series of ten readings with wire horizontal at center—readings taken in groups as before described—gave a mean value of 14.8, while the companion series with the wire removed showed 14.7 mm. galvanometer deflection.

To explain such strong absorption effects optical analogies fail us, and it seems reasonable to refer them to the effects of radiation coming from oscillations induced in the obstructing conductors differing in phase something like a half period from that of the direct radiation. This has been called by Righi secondary radiation.[1] In the reference given he describes an experiment where another resonator similar in dimensions to the one used as receiver was placed at various distances to one side of the receiver and its influence upon the latter noted. This influence was found to vary with the distance, helping in some positions and opposing in others. At distance $= \lambda/2$ a maximum was obtained. This was tested in our apparatus by using a vertical strip of sheet metal whose length and width were equal to the length and diameter of the receiver. When this was placed 6.5 cm. ($= \lambda/2$) to one side of R on a line perpendicular to the axis, it was found to increase the energy received about 12 per cent. Trying non-resonant vertical conductors the long rod of .32 cm. diameter had no measurable influence, but a flat strip 7.5 cm. wide (with its width parallel to the radiation axis) gave a well-defined set of maxima and minima, but with distances between adjacent maxima and minima somewhat less than a half wave-length.

These later experiments and certain quantitative deviations of the diffraction phenomena from what is to be expected from optical analogy seem to find a possible explanation in the influence of secondary radiation from conductors in the vicinity. This suggested a more thorough and systematic study of the influence of neighboring conductors, which has since been made. A description of these experiments, a preliminary account of which has been presented to the Physical Society, will be given in a later paper.

SUMMARY.

1. A study was made of the distribution of radiant energy with several arrangements of electric wave apparatus likely to furnish diffraction effects. Among them were the following cases: radiation passing a thin edge of opaque material, radiation passing through openings of several widths, tested (*a*) for lateral distribution and (*b*) for variation along the

[1] Righi, Die Optik der elektrischen Schwingungen, Chap. II., 1898.

axis. The effect of the screens was to increase the energy received at certain points and some cases analogous to diffraction bands are described.

2. The effect of shifting a pair of "screens" along from one position to another along the radiation axis is described.

3. The effect of a gradual change in the width of an opening through which radiation passed was studied for two points on the axis. Very little energy passed when the opening was less than a quarter wave-length; with openings greater than one and a half wave-lengths more energy appeared beyond than when no screens at all were used.

4. Some resonance and interference results are shown by curves.

5. Large absorption or obstruction effects were found when either a thick or a thin wire was placed at any point along the radiation axis, if its length were parallel to the electric force. A flat strip reduced the intensity even more when placed edgewise than when broadside to the radiation.

6. When such a wire or strip was moved laterally by short steps from a central position strong maxima and minima were shown at the receiver.

7. One of Righi's "secondary radiation" experiments was repeated with the somewhat different apparatus used in this study and his result verified. The suggestion is made that many of the effects observed in these experiments can be explained by the interference of such secondary radiation with that directly received.

OHIO STATE UNIVERSITY,
August, 1912.

THE VELOCITY OF ELECTRONS IN THE PHOTO-ELECTRIC EFFECT, AS A FUNCTION OF THE WAVE LENGTHS OF THE LIGHT.

By David W. Cornelius.

THE object of this investigation is a determination of the potential acquired by alkali metals when illuminated by light of different wave-lengths, and the determination of the velocity of the electrons as a function of the frequency of the incident light. Besides the direct interest of these measurements, the importance of this investigation lies in its connection with the general theory of radiation of the black body as developed by Planck by means of the calculus of probability and thermodynamics. In Planck's theory, which has been well confirmed by the experiments on the radiation of the black body, which also gives good values for the elementary quantities of nature, the elementary oscillator emits the radiant energy not continuously, but intermittently in definite units such that the elementary unit of radiation energy is proportional to the frequency of the light. The equation is

$$E = hn,$$

where E is the energy, n is the frequency and h is a constant of proportionality. On this theory we should expect the positive potential of the photo-electric effect to increase proportionally to n. Some of the first determinations of the velocities of electrons emitted from alkali metals under the action of light of different wave-lengths have been made by Jakob Kunz,[1] who found that the potentials are nearly proportional to the square of the frequencies of the incident light, and that the velocities are almost independent of the temperature and of the intensity of the light. Two other recent papers make further measurements important. J. R. Wright[2] finds that there is a decided maximum in the curve representing potential and frequency in the case of zinc; and R. Pohl and P. Pringsheim[3] find that there are two different photo-electric effects in the alkali metals, the ordinary and the selective effect. They measured, however, the photo-electric currents, and not the equilibrium

[1] J. Kunz, Phys. Rev., Vol. 29, 3, 1909; Vol. 31, 5, 1910.

[2] J. R. Wright, Phys. Rev., Vol. 33, 1, 1911.

[3] R. Pohl and P. Pringsheim, Verhandlungen, d. D. Phys. Gesellschaft, 12, p. 215, 1910; and R. Pohl, Verhandlungen, d. D. Phys. Gesellschaft, 11, 715, 1909; 13, 961, 1911.

potential acquired by the metals. It is therefore important to ascertain whether there are two different effects to be found in this equilibrium potential, whether there is a maximum, or whether the potential increases continuously throughout the range of the visible light. Finally the influence of the temperature on the equilibrium potential has to be determined.

Description of Apparatus and Method.

The arrangement of the essential parts of the apparatus used in this research is shown in Figs. 1 and 2. The source of light is a carbon arc *L*.

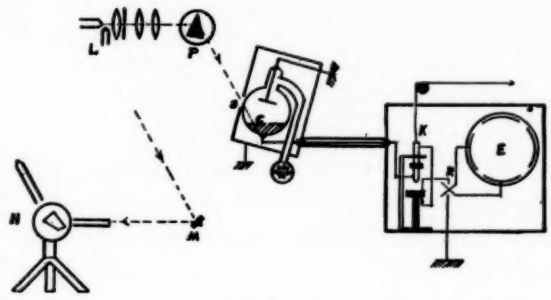


Fig. 1.

The light passes through a slit and system of lenses which gives a beam of parallel light upon the prism *P*, which is capable of rotation, thus providing for an intense source of light, which after passing through the prism, falls upon the slit *S* of a light-tight box containing the photo-electric cell. By rotating the prism *P* any desired wave-length can be made to pass through the slit *S* and fall upon the photo-electric metal *C*. The illuminated electrode *C* of the cell is connected through a key *K* and commutator *R* to a pair of quadrants of a Dolazalek electrometer *E*. Static charges were avoided by having the cell, connections, keys and measuring instrument, which were manipulated from a distance, all inside of earthed conductors. The insulators were made of sulphur and amber plugs. The light which passes through the slit *S* falls upon the metal in the photo-electric cell. The mirror *m* is mounted so that it can be rotated about the axis *ab*. The light of the same wave-length as is incident upon the metal is therefore incident upon the mirror. The light is reflected from the mirror into a direct reading spectrometer *H*, made by Hilger, which was calibrated by means of the sodium lines. This proved to be a very satisfactory arrangement, since the readings of wave-length could be made as the rotating prism was rotated into a

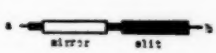


Fig. 2.

desired position. The suspension of the electrometer was a fine quartz fiber which was made conducting by $CaCl_2$ for a portion of the work, and a fine phosphor bronze wire for the remainder. The deflection of the electrometer was read by the image of an incandescent light focused on a millimeter scale at a distance of about five meters from the mirror of the electrometer needle. The electrometer was calibrated by means of a standard Weston cell, resistances and storage battery in the usual way. The calibration of the electrometer is practically a straight line. That is, the deflections were proportional to voltage. The sensibility was .0011 volt per mm. deflection. The electrometer was calibrated several times with practically the same results. The maximum potential acquired by the metal, for incident light of any given wave-length, was determined by the deflection of the electrometer. Various voltages were used on the needle ranging from 40 to 140, depending upon the sensibility desired.

A number of photo-electric cells were constructed, containing different metals. The designs of the cells varied considerably. The determination of the velocity of electrons emitted from the surface of a photo-electric metal in a vacuum tube, when acted upon by light, was attempted by different methods. The stream of negatively charged electrons will be deflected if it passes through a magnetic field. The velocity of the electrons can be calculated by the ratio e/m, where e is the charge and m the mass of the electron, along with H, the magnetic field and the deflection of the stream of electrons. The magnetic deflection method was tried in several tubes. The advantage of this method is that it gives an independent determination of the velocity of the electrons, so that it would be desirable to be able to use it. After repeated trials the magnetic deflection method was given up, since it was found impossible to measure the photo-electric current. The method of measuring the equilibrium potential by means of an electrometer is subject to certain objections which could not be made against the deflection method. Any conductivity along the glass wall or through the vapor of the alkali metal diminishes the final value of the equilibrium potential. Moreover the potential measured by the electrometer is due to the equilibrium potential and partly to contact electro-motive forces or potential differences of any other nature. All those influences could at once be avoided by the magnetic deflection method.

Several cells, which are not shown in a diagram, were unsuccessful. Two rubidium cells showed some interesting features. Exposed to light they showed in the beginning considerable sensitiveness, which decayed rapidly until they did not respond even to intense light. After being left

in the dark for a short time, they would sometimes recover. The photo-electric metal was melted several times, but apparently was not effective in changing the behavior of the cells. The same behavior was observed in a potassium cell. It was put in sunlight for several hours, but this did not result in the cell becoming consistently active. This phenomenon might be called a fatigue, the cause and character of it are still uncertain. Thus we see that this work is attended with many difficulties. There are a number of sources of error. A small leak in the tube, a slight impurity of the photo-electric metal, and its surface conditions, gas developed in the tube due to wax joints or occluded gas in the metals in the tube, static charges on the glass of the tube, imperfect insulation between the electrodes of the cell, enter as factors, rendering this experimental work difficult of execution. In several experiments it has been observed that the photo-electric metal in the dark chamber acquired spontaneously a negative potential, which developed very slowly and arose to quite considerable values. This effect has since been studied by J. W. Wood-

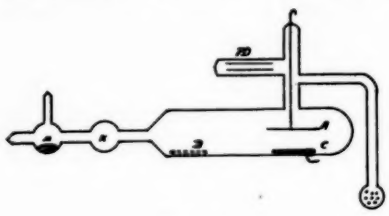


Fig. 3.

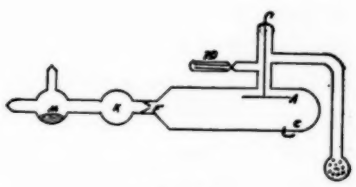


Fig. 4.

row. In those cells which gave the best and most consistent results this negative effect has not been observed. If it existed it must have been so slow that it could not effect the readings to any measurable amount.

The general type of cell which has proved to be the most satisfactory in this investigation is shown in Figs. 3 and 4. The metals used were potassium, potassium "fixed" with hydrogen, cæsium, and cæsium "fixed" with hydrogen. There were only slight variations of the design of the cell to suit the introduction of the different metals, or conditions desired for the cell.

Cæsium was prepared for introduction into the cell, by first drying cæsium chloride by melting it in contact with dry hydrochloric acid gas. Fourteen grams of dry cæsium chloride were mixed with 2.5 grams calcium, placed in an iron boat in a combustion tube of Bohemian glass. When the temperature rises the reaction beween the Ca and the CsCl, or RbCl, becomes quite violent and the calcium and the salts spread out and condense together with the rubidium, or cæsium, in the cooler parts of the combustion tube. To prevent this mixture, a plug of

asbestos and iron wire keep the iron boat in position and allow the rubidium, or cæsium, vapor alone to pass through the plug. The combustion tube is connected by means of sealing wax and glass tubes to the photo-electric cell. After the tube was exhausted the combustion tube was heated gradually until the metal distilled and ran through the connecting tubes into the cell. Rubidium and cæsium were both prepared in the same manner. Potassium was introduced into the bulb *M*, of Figs. 3 and 4, distilled into *K* and poured into *C*. The cells were all exhausted by a Gaede pump. The vacuum in each case was tested by the characteristics of the discharge, from an induction coil between two electrodes in the system. A flame was kept under the charcoal bulb for several hours until the charcoal ceased to develop gas and the pump was able to exhaust the system to a stage of hard Roentgen rays. If the metal introduced was to be left in the pure state without hydrogen, the tube was sealed off from the pump as soon as the metal was distilled and transferred into the final position. However, if the metal was to be "fixed" with the hydrogen, the cell was not sealed off from the pump until later. The hydrogen was introduced by means of palladium, Pd, of Figs. 3 and 4. This metal was used as a cathode in a solution of three parts water. and one part H_2SO_4 When the electric current passes through this cell the Pd absorbs a large amount of hydrogen which is given off again by gently heating the dry metal with a bunsen flame, in Pd of Figs. 3 and 4. When the cell contains a small amount of hydrogen and a discharge passes from the alkali metal to the anode the surface of the alkali metal assumes very intense colors, due to the combination of the metal with hydrogen or else to a colloidal transformation of the metal. This process we may call fixing or forming. In this forming the potassium assumes intense blue or purple colors, while the cæsium exhibits a greenish gray or bronze surface.

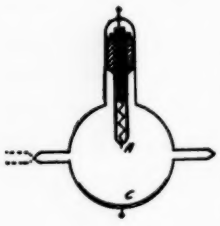


Fig. 5.

A cæsium cell No. 10 without charcoal bulb is shown in Fig. 5. Cæsium is distilled into the bulb at *C* when the electrode *A* is raised, by means of a magnet, in order that no cæsium vapor condense upon the electrode. The diameter of the bulb is about 5 cm., and the distance between *A* and *C* about 2 cm. The data for this cell are given in Table I.

The curve of Table I. is quite similar to that obtained by Jakob Kunz[1] under similar circumstances. The cells which he used did not have any charcoal bulbs attached. Thus it will be noted that this cell which does

[1] J. Kunz, PHYS. REV., Vol. 29, 3, 1909.

not have charcoal bulb attached verifies his work. The variations of this curve from a smooth curve is in a measure due to residual pressure in the cell. This error is reduced almost entirely where charcoal and liquid air are used.

TABLE I.

Cæsium Cell No. 10. $t = 23°$ *C.*

Wave-length. λ $\mu\mu$	Frequency^2. N^2 10^{30}	Volts. v	Wave-length. λ $\mu\mu$	Frequency^2. N^2 10^{30}	Volts. v.
420	.510	1.168	570	.278	0.606
450	.444	1.090	600	.250	0.517
480	.391	1.010	630	.227	0.387
510	.346	0.920	660	.207	0.343
540	.309	0.748	690	.189	0.325

Volts on needle of electrometer 126. $K_{126} = .001202$.

Cæsium cell No. 12 was similar to Fig. 3. The metal was distilled upon an iron plate at *B* of Fig. 3 and moved by means of a magnet to a final position *C*. *A* is an aluminium plate 1 × 3 cm. The distance between electrodes was about 4 mm. The tube was 2 cm. in diameter and 18 cm. long. The cæsium was not "fixed" with hydrogen before the cell was sealed from the pump. The cell was first tried with the metal pure, but was found not to be photo-electric. After three days' effort it was decided to "fix" the metal. It was done in the usual manner. However, since the cell had been previously sealed from the pump, the residual hydrogen in the tube was not pumped out after fixing, but left for the charcoal to absorb when immersed in liquid air. The results of this cell are given in Fig. 6. Liquid air was kept on the charcoal bulb continuously during the series of observations. The time elapsing between the readings for curves 1–4 was about 43 hours.

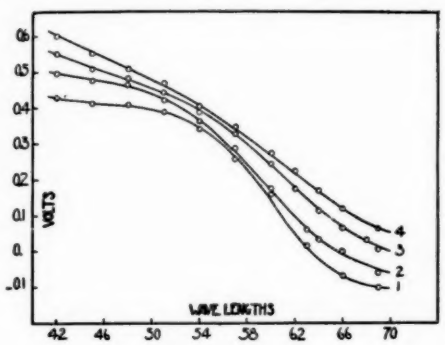


Fig. 6.

The characteristics of the curves change gradually with time. This is due probably to the absorption of the residual hydrogen, by the charcoal. The change seems to indicate that the curves are approaching a form such as would finally make the maximum potential proportional to the square of the frequency. It was impossible to keep liquid air on the cell

continuously, and make observations until such a steady state was reached. It is probable, moreover, that the surface of the metal undergoes slight changes so that the sensitiveness either increases or decreases in the course of time.

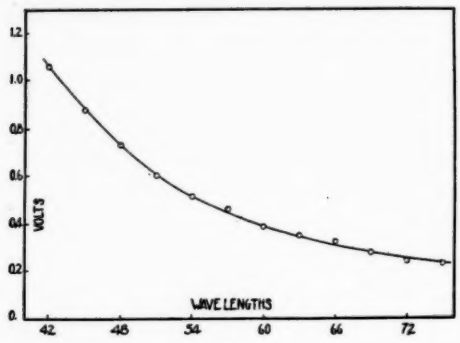


Fig. 7.

Potassium was placed in bulb *M*, Fig. 4, distilled into *K* and poured through funnel *F* into final position *C* in cell No. 4. The surface of the potassium became darkened to a reddish purple color, as the cell was left exposed to the light of the room for six days before the observations, shown in Table II. and curves of Figs. 7 and 8 were taken.

TABLE II.

Cæsium Cell No. 4. $t = 23°$ *C.*

Wave-length. $\mu\mu$	Frequency^2. $\frac{N^2}{10^{30}}$	Volts. v	Wave-length. $\mu\mu$	Frequency^2. $\frac{N^2}{10^{30}}$	Volts. v
420	.510	1.070	600	.250	0.394
450	.444	0.890	630	.227	0.356
480	.391	0.730	660	.207	0.335
510	.346	0.609	690	.189	0.279
540	.309	0.523	720	.173	0.249
570	.278	0.472	750	.160	0.243

Volts on needle of electrometer 127. $K_{127} = .00150$.

These curves are very significant. The curve of Fig. 7 certainly is not a straight line as Planck's law would require, but Fig. 8 shows a straight line within a reasonable error of observation. There is a slight variation in the points near the red end of the spectrum, but that is probably due to the great difficulty of determining those points accurately. It was found that the maximum equilibrium potential was independent of the intensity of the incident light, within the limits of the

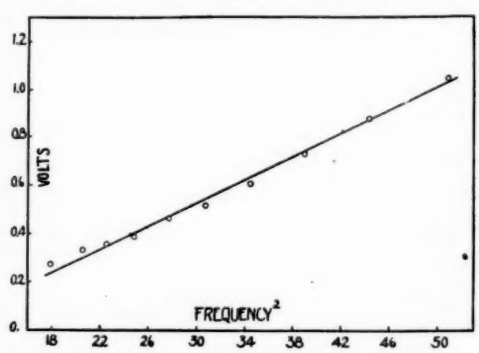


Fig. 8.

full intensity due to the arc, and the intensity of the light due to an incandescent lamp directed upon the cell.

Cell No. 8 was a cell in which the active metal was potassium fixed with hydrogen. It was similar to Fig. 3. The potassium was distilled upon a platinum plate (which had iron attached) at position *B*, Fig. 3, and moved into position *C* by means of a magnet. The electrode distance was about 2 mm. The potassium was fixed in the usual manner, as described before, but was not photo-electric. The movable electrode was moved back to position *B* and the potassium

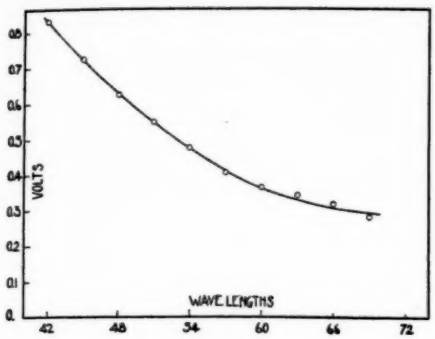


Fig. 9.

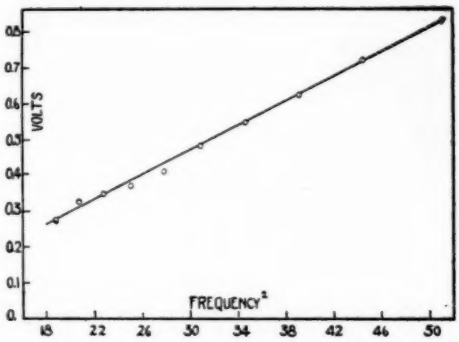


Fig. 10.

melted. When the electrode was removed back to position *C*, it was photo-electric. Observations were made upon the cell for three consecutive days, which indicated a positive potential of the alkali metal when exposed to light. The potential was erratic however. Nine days later consistent observations were made upon the cell as shown in data of Table III., and Figs. 9 and 10.

TABLE III.

Cæsium Cell No. 8. $t = 23°$ C.

Wave-length. $\mu\mu$	Frequency². N^2 10^{30}	Volts. v	Wave-length. $\mu\mu$	Frequency². N^2	Volts. v
420	.510	.835	570	.278	.407
450	.444	.722	600	.250	.361
480	.391	.622	630	.227	.348
510	.346	.537	660	.207	.326
540	.309	.488	690	.189	.266

Volts on needle of electrometer 106. $K_{106} = .00141$.

These plates show the same characteristic properties of the phenomenon. The equilibrium potential is proportional to the square of the

frequency. This is always the case when the metal is in the permanent state of activity.

Cæsium was poured into position *C* in cell No. 13, which was of the form shown in Fig. 4. The surface was "fixed" in the usual manner. The color of the surface was a greenish gray and partly bronze. When

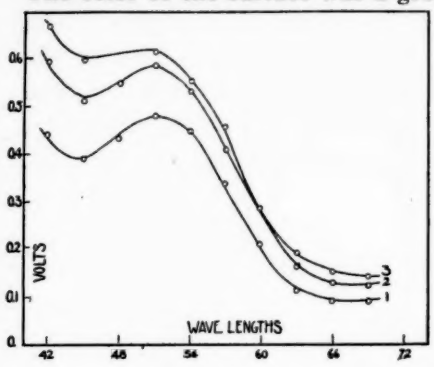


Fig. 11.

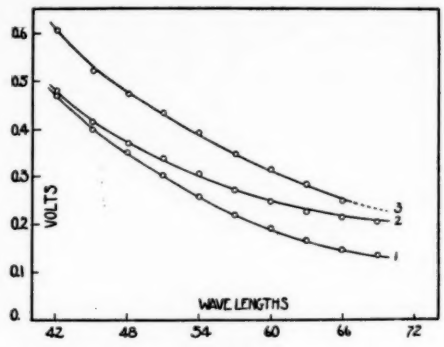


Fig. 12.

liquid air was applied to the charcoal bulb, the metal was not active in the beginning. Violet light was incident upon the cell for an hour, or more, before the electrometer indicated a positive deflection of 510 mm. which is equivalent to .062 vol A set of observations was taken, as recorded in curve 1 of Fig. . he maximum potential was reached very quickly after the metal was connected to the electrometer, but was very difficult to read accurately. Liquid air was kept continuously on the bulb and a series of observations was taken about twelve hours later, as recorded in curve 3, Fig. 11. About nine hours later another series was taken as shown in curve 3 of the same figure. The cell was left undisturbed for four days (due to a lack of liquid air). After this time another series of observations was taken upon the cell as given in Table IV. and curve 1 of Figs. 12 and 13. (The time required for the completion of a series is about two hours.) Then following this series another was immediately taken, Table IV. and curve 2, Figs. 12 and 13. The cell was placed in an ice bath and another set of observations for 0° C. was taken which is recorded in Table IV. and curve 3 of Figs. 12 and 13. Two

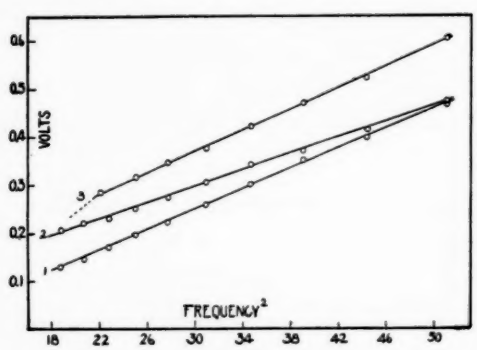


Fig. 13.

days later sets of data were taken for variation in temperature as shown in Table V.

TABLE IV.

Cæsium Cell 13. 4 *days later than Fig.* 11.
λ = wave-length. N^2 = frequency^2. v = volts.

λ	N^2	Time, 3 P. M. Tem., 23° C. v	Time, 6 P. M. Tem., 23° C. v	Time, 10 P. M. Tem., 0° C. v
420	.510	.468	.414	.604
450	.444	.392	.373	.520
480	.391	.355	.368	.470
510	.346	.301	.342	.420
540	.309	.259	.305	.382
570	.278	.220	.272	.345
600	.250	.197	.249	.306
630	.227	.168	.228	.288
660	.207	.145	.220	.248
690	.189	.132	.204	.204

Volts on needle of electrometer 127. K_{127} = .00120.

TABLE V.

Cæsium Cell 13. 2 *days later than Figs.* 12 *and* 13.
λ = wave-length. N^2 = frequency^2. v = volts.

λ	N^2	Tem., 23° C. v	Tem., 40° C. (?) v	Tem., 0° C. v
420	.510	.475	.600	[1]
450	.444	.405	.528	[1]
480	.391	.378	.486	.483
510	.346	.348	.450	.455
540	.309	.318	.414	.420
570	.278	.288	.367	.374
600	.250	.258	.342	.348
630	.227	.242	.294	.298
660	.207	.228	.248	.256
690	.189	.205	.212	.214

Volts on needle of electrometer 127. K_{127} = .00120.

The curves of Fig. 11 indicate a change with time similar to Fig. 6. There seems to be a growth in the sensibility of the cell as well as a smoothing of the curves. The surface of the photo-electric metal seems to undergo a change until it finally reaches a steady state, in which the maximum potential varies as the square of the frequency of the incident light.

[1] Reading of the electrometer uncertain. Possibly rise of the temperature of the cell, poor insulation, and negative effect are the causes of the difficulty. The cell, in this case, became negative in a very few minutes when in the dark.

The set of curves 1 and 2, in Figs. 12 and 13, shows the cell to have reached a steady state. It is interesting to note the development of the cell. There is some suspicion to believe that the sensibility increases within the length of time (two hours) required to make a set of observations. Curve 2 is read from the red to the violet end of the spectrum. Curve 3 for ice temperature is a little uncertain at the end points, the electrometer being difficult to read in that region. All the errors, imperfect insulation, rise of temperature of the cell, etc., tend to lower the points. There is a little uncertainty, therefore, as to the character of curve 3, since the shifting of a few of the end points may be critical in determining the form of the curve.

GENERAL RESULTS.

Let us consider the significance of this experimental work. We see that Planck's law, which states that the maximum potential acquired by the alkali metal should be proportional to the frequency of the incident light, is not corroborated. But, on the other hand, the maximum equilibrium potential of the metal P varies as the square of the frequency n of the light. This relation is expressed by the equation,

$$P = kn^2 + P_0,$$

where k is a factor of proportionality and P_0 a constant. The theory, as developed by J. Kunz,[1] is expressed in the form $E = kn^2$, where E is the energy of the vibrating Faraday tube in the beam of light, k is a factor of proportionality and n the frequency of the light. If light falls upon a photo-electric metal a part of the energy is reflected and a part absorbed, both being proportional to the square of the frequency. In order that an electron may escape from the metal, its kinetic energy must be sufficient to overcome the attraction of the positive charge left behind. An electron, in passing through the metal, may lose some energy by collision with molecules of the metal before it reaches the surface. Let us call this loss of kinetic energy w. Then the electron leaves the surface of the metal with a kinetic energy $E = kn^2 - w$. The metal acquires a positive charge and a maximum potential of P, when the electrons are escaping, which is in equilibrium with the energy of the escaping electrons. If the charge of an electron is e, then $Pe = akn^2 - w$. We are able to calculate the values for the constant w, from the experimental observations. The straight lines of Figs. 8, 10 and 13, which represent the relation of maximum potential and square of frequency, may be expressed in the form

$$P = sn^2 + w.$$

[1] J. Kunz, PHYS. REV., Vol. 29, 3, 1909.

If $P = 0$, then $sn^2 = w$. Thus graphically w is the point where the curve crosses the y axis, if $P = y$ and $x = n^2$. Several values have been calculated from the data which are given in the table below. The method of calculation may be made clear by a sample calculation taken from Fig. 13,

$$s = \frac{P_1 - P_2}{n_1^2 - n_2^2},$$

where P_1 is potential in absolute units for a frequency n_1, and P_2 is the potential in absolute units for a frequency n_2. This gives

$$s = \frac{P_1 - P_2}{n_1^2 - n_2^2} \times 300,$$

where P_1 and P_2 are expressed in volts.

$$s = \frac{(.468 - .132)}{300 \times .51 \times 10^{30} - .18 \times 10^{30}} = .34 \times 10^{-32}$$

for the slope of curve 1, Fig. 13. Applying this value in the formula for the case where $\lambda = 420\mu\mu$

$$w = P_1 - sn_1^2 = .468 - (.34 \times 10^{-32} \times .51 \times 10^{30} \times 300)$$
$$= -.052 \text{ volt} = .174 \times 10^{-4} \text{ abs.}$$
$$e \times w = 4.65 \times 10^{-10} \times 174 \times 10^{-4} = -.0807 \cdot 10^{-12} \text{ ergs.}$$

This represents the energy lost by the electron in reaching the surface or the amount of work necessary to drag an electron to the surface of the metal.

VALUE OF WORK TO BRING ELECTRON TO SURFACE.

TABLE VI.

Metal in Cell.	Cæsium 23° C.	Cæsium 0° C.	Cæsium 23° C., 48 Hrs. Later.	Pot. Cell No. 1.	Pot Cell No. 8 with *H*.
Values of $S \times 10^{-32}$.	.340	.368	.273	.817	.574
	.344	.376	.273	.827	.575
	.338	.369	.272	.822	.563
	.346	.370	.277	.833	.573
	.340	.369	.270	.847	.575
Mean $S \times 10^{-32}$...	.342	.374	.273	.829	.572
Work in volts.....	−.052	+.037	+.057	− .160	−.041
Work in ergs $\times 10^{-13}$	−.807	+.572	+.888	−2.480	−.667

The velocity of an electron emitted from the surface is given by the equilibrium relation $Pe = \frac{1}{2}mv^2$, where P is the maximum potential, e

the charge of the electron, m its mass and v its initial velocity of emission from the surface. Hence

$$v = \sqrt{2\frac{e}{m}P}$$

gives the formula for the calculation of the velocity where e/m is 1.77×10^7 and P has values as determined by observations.

VALUES OF INITIAL VELOCITY OF ELECTRONS.

TABLE VII.

Cell.	λ in $\mu\mu$.	P Abs. Units.	$V = \sqrt{2\frac{e}{m}P}$, $\frac{\text{cm}}{\text{sec}}$.
Pot. No. 4.....	420	1.09×10^8	6.21×10^7
Pot. No. 4.....	690	$.266 \times 10^8$	0.97×10^7
Pot. No. 4.....	750	$.233 \times 10^8$	0.91×10^7
Pot. No. 8.....	420	$.835 \times 10^8$	5.45×10^7
Pot. No. 8.....	690	$.266 \times 10^8$	0.97×10^7
Cæsium No. 13 with H	420	$.472 \times 10^8$	4.09×10^7
Cæsium at 23° C.....	690	$.780 \times 10^8$	0.78×10^7
At ice.....	420	$.600 \times 10^8$	4.61×10^7
At 0° C.....	690	$.177 \times 10^8$	0.79×10^7

Planck's law as expressed by the equation,

$$E = hn = \tfrac{1}{2}mv^2 + C, \qquad (1)$$

which may be written

$$hn = Pe + C,$$

gives the right order of magnitude for the velocity of the electron although it does not express the relation of the velocity of the electrons as a function of the frequency of the incident light. h has a value of 6.548×10^{-27} ergs sec. The velocity is given by the equation (1) $v = \sqrt{2hn/m}$. For light of wave-length $420\mu\mu$, $n = 0.715 \cdot 10^{15}$. The mass of the electron is 8.7×10^{-28} grams. These values applied to the formula give

$$v = \sqrt{\frac{2hn}{m}} = \sqrt{\frac{2 \times 6.55 \times 10^{-27} \times .715}{8.7 \times 10^{-28}}} = 10.3 \times 10^7 \text{ cm. sec.}$$

This compares well with the experimental values. However, the experimental values for the maximum potential acquired by the metal, for light of different frequencies, are different from those calculated by means of the equation of Planck,

$$hn = Pe + C,$$

where h is 6.5×10^{-27}, e is 4.65×10^{-10} and C is 2.4×10^{-12}. This varia-

tion of the experimental results from Planck's law is made clear by a set of calculations for potassium cell No. 8, which is shown in Table VIII.

TABLE VIII.

ν 10^{15}	Potential V_1 Calculated by Planck's Law, Volts.	Potential Observed V_2.	Difference $V_1 - V_2$ Volts.
.435	.266	.266	
.455	.354	.326	.028
.476	.438	.348	.090
.500	.541	.361	.180
.527	.654	.407	.247
.556	.781	.488	.293
.588	.909	.537	.372
.625	1.063	.622	.441
.666	1.242	.722	.520
.714	1.440	.835	.605

The work done in expelling an electron can be computed. The work done in moving an electron of charge e_1 (which must be equal to e_2 or a multiple of e_2) through a distance dr is

$$\begin{array}{cc} r & dr \\ \hline e_1 & e_2 \end{array}$$

$$dw = \frac{e_1 e_2}{r^2} dr.$$

The work done in moving the electron to infinity (*i. e.*, beyond the field of attraction of the atom) is:

$$w = e_1 e_2 \int_R^\infty \frac{dr}{r^2} = -\frac{e_1 e_2}{r}\Big|_R^\infty = \frac{e_1 e_2}{R},$$

where R is the radius of the atom. Now

$$w = \frac{e_1 e_2}{R} = \tfrac{1}{2} m_2 v^2.$$

Hence

$$v = \sqrt{\frac{2 e_1 e_2}{R m_2}}.$$

This is the velocity necessary that the electron may escape from the metal.

If an electron revolves about the positive atom it has kinetic energy due to its rotation. The velocity of the electron is determined by the equilibrium of the centrifugal and centripetal forces. If the electron has a mass m_2, a charge e_2, linear velocity v and the positive atom has a

mass m_1, charge e_1 (which must equal e_2 or a multiple of e_2) and a radius R; then

$$\frac{1}{2}\frac{m_2v^2}{R} = \frac{1}{2}\frac{e_1e_2}{R^2},$$

if the revolution is in a circle, which is the case when m_1 is large as compared to m_2. This gives

$$\text{K.E.} = \tfrac{1}{2}m_2v^2 = \frac{1}{2}\frac{e_1e_2}{R}.$$

F. A. Lindemann[1] has used this formula in his determination of the wave-length for which the selective photo-electric current reaches a maximum. Thus the work that has to be communicated to the electron by the incident light is the difference between the total work done in the expulsion of an electron and its energy due to rotation, viz.,

$$w - \text{K.E.} = \frac{1}{2}\frac{e_1e_2}{R}.$$

The values of the velocity, computed by this deduction, are near the values found by experiment. Suppose that

$$e_1 = \text{charge of the atom} = e_2 = 4.65 \times 10^{-10}$$

$$\frac{e_2}{m_2} = 5.35 \times 10^{17}.$$

R_k = radius of potassium atom = 2.37×10^{-8} cm. used by Lindemann.[2]

$$\therefore v_k = \sqrt{\frac{2 \times 4.65 \times 10^{-10} \times 5.35 \times 10^{17}}{2.37 \times 10^{-8}}} = 4.58 \times 10^7 \frac{\text{cm.}}{\text{sec.}}$$

= velocity of the electron from potassium atom.

The atomic volumes vary as the cubes of the atomic radii, so we have

$$R_{cs}^3 = R_k^3 \frac{a_{cs}}{a_k},$$

which determines the value of R_{cs} when R_k is known. $R_{cs} = 2.75 \times 10^{-8}$ cm. Then

$$v_{cs} = \sqrt{\frac{2 \times 4.65 \times 10^{-10} \times 5.35 \times 10^{17}}{2.75 \times 10^{-8}}} = 4.25 \times 10^7 \frac{\text{cm.}}{\text{sec.}}$$

= velocity of electron from cæsium in order that it may escape.

[1] F. A. Lindemann, Verh. der Deutschen Phys. Gesell., 13, No. 12, 1911.
[2] F. A. Lindemann, Verh. Deutschen Phys. Gesell., 13, No. 12, 1911.

By means of the formula,

$$w_L = \frac{e_1 e_2}{2R},$$

the work necessary to expel an electron from the metal can be computed. Using values as in the above calculation, we get for the work which needs to be supplied by the incident light to expel an electron, in case of potassium,

$$w - \text{K.E.} = \frac{1}{2}\frac{e_1 e_2}{R} = \frac{1}{2}\frac{(4.65 \times 10^{-10})^2}{2.47 \times 10^{-8}} = 4.57 \times 10^{-12} \text{ ergs.}$$

For cæsium,

$$w - \text{K.E.} = \frac{1}{2}\frac{(4.65 \times 10^{-10})^2}{2.75 \times 10^{-8}} = 3.82 \times 10^{-12} \text{ ergs.}$$

The electron theory gives a means of calculating the time required for an electron to be emitted photo-electrically. The differential equation for the condition of resonance of the electron, when acted upon by an electrical wave, due to the vibration of the incident light, is

$$m\frac{d^2x}{dt^2}\frac{dx}{dt} + a^2x\frac{dx}{dt} + \frac{2}{3}\frac{e^2}{C}\left(\frac{d^2x}{dt^2}\right)^2 = E_0 \cos \phi t \frac{dx}{dt}.$$

This may be written

$$\frac{dE}{dt} + \frac{2}{3}\frac{e^2}{C}\left(\frac{d^2x}{dt^2}\right)^2 = E_0 \phi t \frac{dx}{dt}. \tag{1}$$

E is the energy of the vibrating electron, e its charge, C the velocity of light, E_0 the amplitude of the electric force in the light vector, times e, ϕ the frequency $= 2\pi n$, and x the displacement from the position of equilibrium. A particular solution gives $x = h \cos(\phi t - \alpha)$.

$$\frac{dx}{dt} = -h\phi \sin(\phi t - \alpha), \quad \text{tg } \alpha = \frac{2}{3}\frac{e^2}{C}\frac{\phi^3}{a^2 - m\phi^2},$$

$$h = \frac{E_0}{m\sqrt{(\phi_0^2 - \phi^2)^2 + \frac{2}{3}\left(\frac{e^2\phi^3}{Cm}\right)^2}},$$

where ϕ_0 is the frequency of the applied electrical force and ϕ is the frequency of the electron's vibrations. Introducing these values in the equation (1) gives

$$dE + \frac{2}{3}\frac{e^2}{C}\left(\frac{d^2x}{dt^2}\right)^2 dt = -\frac{E_0^2 \phi \cos \phi t \sin(\phi t - \alpha) dt}{m\sqrt{(\phi_0^2 - \phi^2)^2 + \left(\frac{2}{3}\frac{e^2}{C}\frac{\phi^3}{m}\right)^2}}.$$

If the electron is considered to have a unit charge then the work done in one revolution, in time T, is

$$E_T - E_0 + \frac{2}{3}\frac{e^2}{C}\int_0^T \left(\frac{d^2x}{dt^2}\right)^2 dt = -\frac{E_0^2\phi \int_0^T \cos\phi t \sin(\phi t - \alpha)dt}{m\sqrt{(\phi_0^2 - \phi^2)^2 + \left(\frac{2}{3}\frac{e^2}{mC}\phi^3\right)^2}}.$$

To integrate $\int_0^T \cos\phi t\,(\sin\phi t\cos\alpha - \cos\phi t\sin\alpha)\,dt$, put $\phi t = x$, $\phi dt = dx$, $dt = dx/\phi$. Then the integral becomes $-\sin\alpha\cdot T/2$.

Hence,

$$E_T - E_0 + \frac{2}{3}\frac{e^2}{C}\int_0^T \left(\frac{d^2x}{dt^2}\right)^2 dt = \frac{E_0^2\phi\sin\alpha T}{2m\sqrt{(\phi_0^2 - \phi^2)^2 + \left(\frac{2}{3}\frac{e^2}{mC}\phi^3\right)^2}}.$$

Since $\phi T = 2\pi$,

$$E_T = \frac{E_0^2\sin\alpha\pi}{m\sqrt{(\phi_0^2 - \phi^2)^2 + \left(\frac{2}{3}\frac{e^2}{mC}\phi^3\right)^2}}$$

is the work done in one revolution or complete vibration of the light vector considering the moving charge as unity. If the electron is in resonance with the light vector, then $\phi_0 = \phi$ and $\alpha = \pi/2$. Hence,

$$E_T = \frac{E_0^2\sin\alpha\pi}{\frac{2}{3}\frac{e^2}{C}\phi^3} \text{ becomes } \frac{E_0^2\pi}{\frac{2}{3}\frac{e^2}{C}\phi^3}.$$

If the charge of the electron be taken as e instead of unity, as in the above consideration, the work dw done by the electric force E_0 is

$$dw = E_0 e dx.$$

Therefore,

$$E = \frac{(E_{0cs}e_{cs})^2\pi}{\frac{2}{3}\frac{e_{em}^2}{C}\phi^3} = \frac{3}{16}\frac{E_0^2\lambda^3}{\pi^2}.$$

The values for $E_0 = 0.003$ E.S.U. (estimated) and $\lambda = 420\mu\mu = 4.2 \times 10^{-5}$ cm., when substituted in the above equation, give

$$E = \frac{3}{16}\frac{(.003)^2(4.2\times10^{-5})^3}{(3.1416)^2} = 1.3\times10^{-20} \text{ ergs.}$$

This is the work supplied to the electron by one vibration of the light vector. It was found by means of the previous deduction that the work

necessary to liberate an electron is 4.2×10^{-12} ergs. Thus we get the number of vibrations necessary to liberate an electron which is

$$\frac{4.2 \times 10^{-12}}{1.3 \times 10^{-20}} = 3 \times 10^{8}.$$

Therefore, the length of time to liberate an electron is that required for 3×10^{8} vibrations of the light vector. For $\lambda = 420\mu\mu$, the time is

$$t = \frac{\lambda}{C} = \frac{4.2 \times 10^{-5}}{3 \times 10^{10}} = 1.4 \times 10^{-15} \text{ sec.}$$

Hence the time for 3×10^{8} vibrations is $3.10^{8} \times 1.4 \times 10^{-15} = 4.2 \times 10^{-7}$ second.

Thus the time required for the liberation of an electron from a molecule by the incident light is very small. A conception of this period of time, however, can be obtained in terms of the distance, traversed by light in this interval, which is $3 \times 10^{10} \times 4 \times 10^{-7} = 120$ meters.

Summary.

The principal results of this investigation are as follows:

1. A small amount of residual gas in the photo-electric cell influences its behavior.

2. The surface conditions of the alkali metal has a very large influence upon the photo-electric effect; which may be constant from the beginning or increase or decrease in the course of time.

3. It takes, as a rule, some time for a cell to reach a steady state of sensibility.

4. The attempts to measure the velocities of the electrons by means of the magnetic deflection have been unsuccessful.

5. The results obtained indicate that the equilibrium potential depends to some extent upon the temperature of the metal, above zero degrees; below zero degrees the equilibrium potential seems to be independent of the temperature. Further experiments upon this point are necessary.

6. The relation of the velocity of the electrons and the frequency of the incident light is the same for: pure potassium, potassium "fixed" with hydrogen, cæsium and cæsium "fixed" with hydrogen. And the relation between the equilibrium potential and the frequency of the incident light is the more constant the nearer the metal approaches the permanent state of sensitiveness.

7. The theoretical and calculated values of the initial velocity of the electrons are both of the order of 10^{7} cm. per second.

8. The theoretical value of the time required for the expulsion of an

electron due to the resonance effect of the incident light is of the order of 10^{-7} second.

9. The equilibrium potential of the electrons in the photo-electric effect varies directly as the square of the frequency of the incident light. Planck's law, according to which the units of the electromagnetic energy are proportional to the frequency is not confirmed by the results of this investigation.

10. While the theory of resonance shows that a beam of light may supply a sufficient amount of energy for the electron to escape it cannot satisfactorily account for the essential fact that the velocity of the electrons escaping from the metal is proportional to the frequency.

The author takes great pleasure in acknowledging his indebtedness to Professor A. P. Carman for the facilities for this investigation and to Professor Jakob Kunz, both for his general supervision of the work and for many valuable suggestions.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS.
May 6, 1912.

ON THE COMPARATIVE ABSORPTION OF γ AND X RAYS.

By S. J. Allen and E. J. Lorentz.

Introduction.

In the April, 1912, number of this magazine one of the authors published a number of results on the absorption of γ rays by matter. These results were confined mainly to the hard γ rays after they had passed through different thickness of matter. The results showed: that the γ rays are not homogeneous; that the absorption per unit of mass is a function of the atomic weight, increasing in a fairly uniform manner with the heavier atoms; that the hardening of the rays by passing through matter depended on the atomic weight, and was large for the heavy atoms. In the case of compound substances the absorption depends on the kind and relative number of atoms present, and is independent of the chemical grouping.

The present paper is a continuation, and deals with the soft γ rays, and X rays of various degrees of hardness. An attempt is made to get very hard X rays so as to bridge over the gap between them and the softest γ rays and to compare their absorptive characteristics side by side.

While not as successful as was hoped the authors obtained X rays whose coefficients of absorption were only about four times those for the softest γ rays. Aluminum was used as the standard of comparison.

The work of Benoist has shown that the absorption of X rays is some function of the atomic weight, increasing in general with heavy atoms, but showing many anomalous results. A number of atoms, notably that of silver, exhibit quite different characteristics for hard and soft rays.

The very important results of Barkla and others on the absorption of X rays have definitely proved the existence of true secondary X rays, which he has termed the fluorescent or characteristic X rays. These secondary rays are produced by the absorption of the primary rays only when the hardness of the primary is greater than that which the particular substance under consideration is capable of producing. These secondary rays are homogeneous, and their hardness increases with the atomic weight of the bombarded substance. They are not dependent upon the quantity, but only upon the quality, of the exciting primary radiation.

Besides this true secondary radiation from the bombarded substance there are a large number of corpuscles liberated, and also a certain amount of scattered primary rays.

No true secondary radiation of the same nature as this characteristic X radiation has been noticed for the γ rays, and the anomalous results for X rays is also lacking in any marked degree.

The evidence at present, to decide whether the ionization in a closed vessel exposed to X rays is caused in any part by the direct influence of the primary rays or wholly by the secondary effect of the corpuscles, does not seem conclusive.

EXPERIMENTAL ARRANGEMENTS.

The apparatus used in these experiments was quite simple and can be quickly described. That used for the γ rays is shown sketched in Fig. 1. It consists of a gold leaf electroscope A of brass sides and top, mounted above and directly over the pole pieces N and S of a powerful electro-magnet. The electroscope was surrounded on the sides by a thick cylinder of cast iron B. The base was closed by a thin sheet of iron. The electroscope was operated and read in the usual manner.

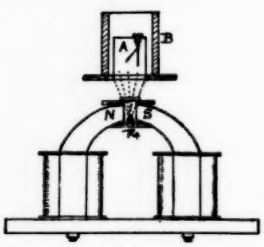


Fig. 1.

The radium Ra covered by only the thin glass walls of the containing tube was placed at the base of the pole pieces and halfway between them at the top of the pole pieces was placed the absorbing layer.

The stream of γ and β rays, which left the radium and passed upward between the pole pieces towards the electroscope, would by the time it reached it consist almost wholly of γ rays, since the β rays would be completely turned aside by the powerful magnetic field.

The iron shield around the electroscope protected to a considerable degree the interior from the magnetic field, so that the ionization would not be complicated by the curved trajectories of the electrons set free by the γ rays from the walls of the electrocsope.

The emergent electron radiation from the various absorbing layers would also be turned aside to a great extent by the magnetic field.

The sides of the pole pieces and all neighboring metallic masses were covered with cardboard to cut down as far as possible the secondary radiations.

By these arrangements the ionization inside the electroscope could be taken as a measure of the intensity of the γ rays.

The stream of γ rays reaching the electroscope would thus be quite pure and as soft as it was possible to obtain from radium in radioactive equilibrium.

Experiments were first made to determine how strong a magnetic field was necessary to completely eliminate the electron radiations. After a strength of field of 4,500 C.G.S. units was reached it was found that further increase of field did not materially reduce the ionization, and all experiments were made at this strength.

The distance between the radium and electroscope was about 15 cm. and the absorbing layer was placed about half way.

In the case of the X rays several arrangements were used. For the soft rays, where the tube could be run in a steady manner, the gold leaf electroscope, and also the radium balanced electrometer were used, and fairly consistent results obtained.

For the medium and very hard rays these methods were no longer of any use, since the variations in the running of the tubes were so abrupt and rapid as to prevent time or "rate of leak" readings being taken with any consistency.

In order to eliminate as far as possible the effect of this rapid variation in the tube a balance method was used.

The principle of this can be readily seen by an examination of Fig. 2. A rectangular brass box was divided into two compartments *A* and *B* by an insulated brass plate *a*. The chamber *A* contained an insulated fixed plate *b*, and the chamber *B* an insulated plate *c* which could be moved in and out, changing the volume of *B* at will. The plate *a* was connected to one pair of quadrants of the electrometer, the other pair being earthed. Plate *b* was connected to one pole of a battery and plate *c* to the other, the middle of the battery being earthed.

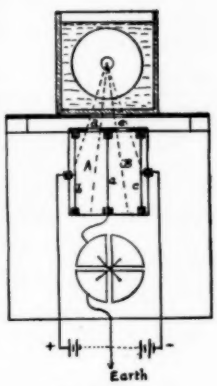


Fig. 2.

The top of the brass box was covered with a lead plate of sufficient thickness to absorb all X rays. Two openings, *d* and *e*, were cut in this plate, one opening into chamber *A*, and the other into *B*.

The box, electronometer, keys, and all connections, were completely enclosed in a tin box to shield them from all electrostatic influences. The top of this box was covered by a thick lead screen with an opening cut to allow the rays to pass through. Above this was placed a large wooden box, filled with oil, in which the X ray tube was immersed to prevent, punctures, and sparking between the terminals.

The electric potential was furnished by a large static influence machine, capable of giving in dry weather a spark of 30 inches or more. The wire leads from the machine to the tube had to be enclosed in glass tubes to prevent brush and spark discharges.

The X-ray tube was connected by a two-way cock to a Gaede mercury pump, and to an ordinary mercury hand pump. By means of the two pumps a low vacuum could be maintained.

The passage of current through the tube at very high degrees of hardness was found by the authors to be very irregular, and intermittent. In order to start the tube a very high potential was necessary, but after it had started the potential dropped to a much lower value. Sometimes the tube would flicker back and forth very rapidly, the walls instead of having a uniform fluorescence would be covered with a large number of brilliant green spots with dark spaces between. From the experience of the authors it would seem very difficult to get an X-ray tube to run much harder.

The method of making an absorption test was as follows: The tube being started, and the windows *d* and *e* opened, the plate *c* was moved back and forth until the electrometer on being separated from earth indicated a balance. The layer of material to be investigated was then placed over the window *e*, thus disturbing the balance. Various layers of aluminum were then placed over the window *d* until the balance was restored. In this case the rays absorbed by the given substance were equal to that absorbed by the aluminum, and the necessary thickness of aluminum was taken as the "equivalent" or standard. This of course gives only the comparative absorption in terms of the aluminum standard and not the absolute.

By closing the chamber *B* completely, and connecting the quadrants to the radium standard, an absolute measurement of the absorption by aluminum could be made, provided the tube would run steady enough. For the very hard rays this was not found to be practical, and so only the comparative absorptions could be obtained.

RESULTS FOR SOFT γ RAYS.

In Table I. are expressed the results for soft γ rays, column one giving the substance, column two the amount of rays transmitted, column three the value of the coefficient of absorption, calculated in the ordinary manner, divided by the density, and column four the relative absorption in terms of aluminum.

By an examination of this table it will be seen that the results are the same as already obtained for the hard rays, but very much more marked. Elements below aluminum in atomic weight have practically the same absorption per unit mass, but above, the divergence increases rapidly with the atomic weight.

The decrease of λ/d with the absorption for the light atoms is not very

marked, but with increasing atomic weight it becomes quite large. It would seem from these results that the γ rays are not homogeneous, but it is possible that they exist in groups when we are dealing with radium in radioactive equilibrium.

Moseley and Makower in a paper in the Philosophical Magazine for February, 1912, show that there is a soft γ radiation from Ra *B*, for which they calculated in an approximate manner λ for lead to be from 4 to 6 cm. This would give $\lambda/d \times 100$ values from 35 to 53.

The largest value for lead obtained by the authors was about 21, which is of the same order of magnitude. This value of course contains some hard rays which would tend to make it smaller.

In the following table are also shown results for pure liquids and aqueous solutions.

TABLE I.

Substance.	Transmission in Per Cent.	$\frac{\lambda}{d} \times 100$.	Relative Absorption Al=1.
Carbon	90	5.46	.97
Carbon	85	5.56	.99
Carbon	80	5.61	1.01
Aluminum	90	5.61	1.0
Aluminum	85	5.61	1.0
Aluminum	80	5.55	1.0
	75	5.45	1.0
Sulphur	90	6.02	1.07
Sulphur	85	5.67	1.01
Sulphur	80	5.52	.99
Wrought iron	90	6.76	1.20
Wrought iron	85	6.30	1.12
Wrought iron	80	5.83	1.05
Copper	90	9.86	1.76
Copper	85	8.29	1.48
Copper	80	7.26	1.31
Zinc	90	10.53	1.88
Silver	90	11.1	1.98
Silver	75	8.42	1.54
Tin	90	12.0	2.14
Tin	80	9.71	1.73
Tin	75	8.96	1.64
Mercury	90	17.2	3.07
Mercury	80	14.3	2.58
Mercury	75	12.9	2.37
Lead	90	20.7	3.69
Lead	85	17.9	3.19
Lead	80	15.1	2.72
Bismuth	90	24.9	4.44
Bismuth	85	20.7	3.69
Bismuth	80	17.5	3.15

Pure Liquids.

Water, H_2O	90	5.85	1.04
	85	6.01	1.07
	80	6.19	1.11
Ether, $(C_2H_5)_2O$	90	5.8	1.04
	85	6.0	1.07
	80	6.2	1.11
Amyl alcohol, $C_5H_{11}OH$	90	6.0	1.07
	85	6.1	1.08
	80	6.3	1.14
Benzene, C_6H_6	90	5.8	1.04
	85	6.0	1.07
	80	6.2	1.11
Amyl bromide, $C_7H_{11}Br$	90	6.8	1.21
	85	6.5	1.16
	80	6.1	1.08

Aqueous Solutions.

Substance.	Transmission in Per Cent.	$\frac{\lambda}{d}\times 100$.	Relative Absorption Al=1.
Chloroform, $CHCl_3$	90	6.2	1.11
	85	6.1	1.08
	80	6.0	1.07
Potassium bromide, KBr	90	6.9	1.23
	85	6.8	1.21
	80	6.7	1.20
Potassium chloride, KCl	90	5.7	1.02
	85	5.9	1.05
	80	6.1	1.09
Potassium iodide, KI	90	11.3	2.01
	85	9.7	1.73
	80	8.9	1.60
Cadmium iodide, CdI_2	90	8.7	1.55
	85	8.4	1.49
	80	7.9	1.42
Copper chloride, $CuCl_2$	90	6.4	1.14
	85	6.3	1.12
	80	6.2	1.11

For the liquids containing only C, H, and O, the values of λ/d are the same, thus agreeing with the additive law. It will be noticed that there is an apparent increase in the values of λ/d with increasing absorption. The same is true for pure carbon but to a less extent. Whether this is a true increase, or due to the thick layers necessary in the case of these light liquids to get appreciable absorption, is not clear.

Whenever there is present an atom of greater atomic weight than that of aluminum, it can be distinguished by the increased absorption.

The results for the aqueous solutions agree closely with the additive law.

The results for the pure metals are shown plotted in Fig. 3, the ordinates being the absorption per unit mass, and the abscissæ atomic weights.

The resulting curves are fairly smooth, but not uniform, and indicate undue absorption for elements around Cu and Zn.

All these results for the soft γ rays can be considered as an extension of those for the hard γ rays of the previous paper, and are in general agreement with them.

RESULTS FOR X RAYS.

In Table II. are expressed the results obtained for X rays by the "rate of leak" method, column four giving the absolute absorption per unit of mass, and column five the relative absorption in terms of aluminum. These results extend from a very soft ray to a very hard one.

In order to bring out all the characteristics a very careful examination of the table is necessary, but it can be said in general that the absorption per unit mass increases with the atomic weight, being very marked in the case of soft rays, and less so for the hard ones.

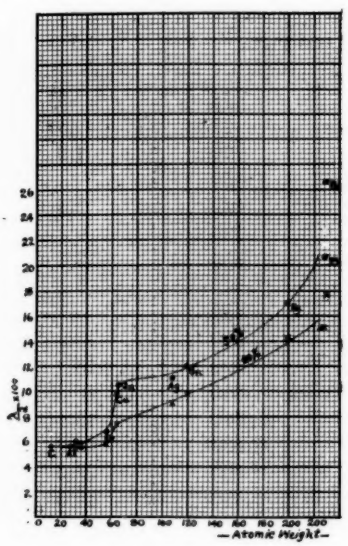


Fig. 3.

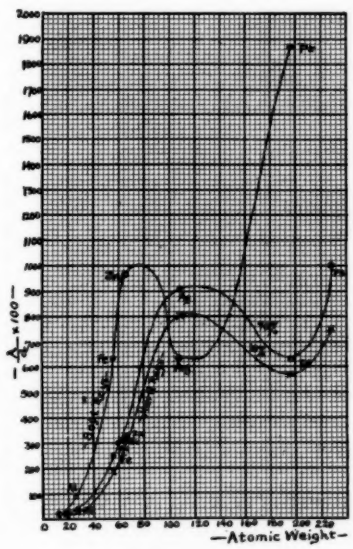


Fig. 4.

There are however exceptions to this, notably in the case of silver. For very soft rays, silver is a much poorer absorber than platinum, but for hard rays it is considerably better. Carbon also shows large variations, for soft rays it is very transparent compared with aluminum, but for hard rays it is much nearer to it.

TABLE II.

Very Soft Tube, about 4,000 *Volts Potential Difference.*

Substance.	Transmission in Per Cent.	$\frac{A}{d} \times 100$.	Relative Absorption.
Carbon	30	250	.12
Aluminum	16	1,600	1.00
Silver	30	7,600	5.0
Soft Tube, about 13,000 *Volts Potential Difference.*			
Carbon	70	74	.11
	50	68	.12
	20	59	.14
	10	55	.16
Aluminum	70	670	
	50	560	1.0
	20	410	
	10	340	
Iron	6.7	2,020	6.5
Zinc	4.8	2,910	10.0
Silver	50	3,400	6.0
	20	2,400	6.0
	10	2,100	6.2
Platinum	5	5,600	20.0
Medium Hard Tube, about 35,000 *Volts.*			
Carbon	42	25	.35
	25	23	.33
Aluminum	59	90	1.0
	17	50	
Sulphur	18	120	2.4
Iron	70	580	–
	49	570	8
	23	540	9
	10	450	–
Nickel	9	520	13
Copper	11.5	590	15
Zinc	42	860	16
	24	710	15
Silver	26	1,430	23.0
	14	1,390	28.0
Platinum	46	1,450	19.0
	26	1,260	21.0
Hard Tube, about 100,000 *Volts.*			
Carbon	70	24	.60
	50	22	.58
	20	19	.63
	10	19	.68
Aluminum	70	40	
	50	38	1.0
	20	30	

TABLE II.—*Continued.*

Su stance.	Transmission in Per Cent.	$\frac{\lambda}{d} \times 100$.	Relative Absorption.	
Aluminum	10	28		
Sulphur	40	67	1.7	
Iron	70	253	6.3	
	50	240	6.3	6.5
	20	203	6.7	
	10	190	6.8	
Nickel	70	300	7.5	
	50	276	7.3	8.1
	20	256	8.5	
	10	256	9.1	
Copper	70	315	8.0	
	50	281	7.4	8.2
	20	258	8.6	
	10	247	8.8	
Zinc	70	333	8.3	
	50	319	8.4	8.8
	20	278	9.2	
	10	264	9.4	
Silver	70	910	22.7	
	50	830	21.8	24.3
	20	800	26.6	
	10	740	26.4	
Platinum	70	630	15.6	
	50	620	16.3	17.7
	20	570	19.0	
	10	560	20.0	
Tinfoil	70	1,000	25.0	
	50	890	23.4	2.5
	20	750	25.0	
	10	740	26.4	

It can be seen clearly that as the rays get harder the difference in the absorption per unit mass for the different elements gets less and less. This fact would be in confirmation with the hypothesis of the identity of γ and X rays.

In Fig. 4 results are plotted for hard and soft rays, and serve to illustrate the very anomalous character of silver.

In Table III. are shown results obtained by the balanced method described above, column four giving the relative absorption per unit mass in terms of aluminum. This relative absorption is calculated from the formula, $t_a d_a / t_x d_x$, where t_x and d_x are the thickness of layer and density of any substance, while t_a is the equivalent thickness of aluminum to produce equal absorption, and d_a the density of aluminum.

It can be seen from the table that the results are practically the same as those by the "rate of leak" method, and wherever the hardness of the ray is the same they are in close agreement.

TABLE III.

Soft Tube, about 10,000 *Volts.*

Substance.	Thickness in Cms.	Equivalent of Al.	Relative Absorption.	
Carbon	.64	.052	.14	
	1.28	.103	.14	
	1.92	.159	.15	
	2.56	.240	.16	
	3.20	.302	.19	
Sulphur	.635	.920	1.95	
Iron	.0160	.437	9.4	9.3
	.325	.870	9.2	
Nickel	.0495	1.670	10.3	
Zinc	.0145	.436	11.1	
	.0287	.933	12.4	12.4
	.0472	1.730	13.9	
Silver	.0040	.0860	5.52	
	.0120	.3563	7.62	
	.0240	.8640	8.26	
	.0280	1.004	9.57	
Platinum	.0025	.380	18.6	
	.0050	.761	19.1	19.6
	.0075	1.230	21.0	
Tinfoil	.0049	.450	22.9	
	.0098	1.000	25.5	24.9
	.0147	1.550	26.3	

Tube about 25,000 *Volts.*

Carbon	.64	.054	.15	
	1.28	.125	.17	
	1.92	.185	.17	
	2.56	.266	.18	
	3.20	.344	.19	
	4.48	.513	.20	
	5.76	.706	.22	
Sulphur	.635	.918	1.77	
Iron	.0160	.361	7.8	
	.0325	.734	7.8	7.9
	.0620	1.440	8.0	
Nickel	.0495	1.650	10.2	
Copper	.0420	1.420	10.3	
Zinc	.0185	.529	10.6	
	.0327	.990	11.2	11.0
	.0472	1.440	11.3	
Silver	.0040	.102	6.6	
	.0120	.412	8.8	
	.0160	.650	10.4	
	.0240	.986	10.5	
	.0280	1.257	11.5	
Platinum	.0025	.390	19.6	
	.0050	.785	19.5	19.8

TABLE III.—*Continued.*

Substance.	Thickness in Cms.	Equivalent of Al.	Relative Absorption.	
Platinum	.0075	1.200	20.0	19.8
	.0100	1.618	20.2	
Tinfoil	.0049	.514	26.1	26.5
	.0098	1.057	26.9	

Tube about 50,000–60,000 *Volts.*

Carbon	1.28	.1296	.18	
	2.56	.318	.22	
	3.84	.540	.25	
	5.12	.842	.29	
Sulphur	.635	1.010	2.1	
Iron	.0160	.412	8.9	8.7
	.0325	.827	8.8	
	.0620	1.555	8.7	
	.0945	2.330	8.5	
Nickel	.0495	1.865	11.6	
Copper	.0425	1.511	10.8	
Zinc	.0142	.558	14.5	13.9
	.0287	1.077	13.9	
	.0427	1.534	13.3	
Silver	.0040	.133	8.6	
	.0120	.663	14.2	
	.0160	1.004	16.1	
	.0240	1.860	19.9	
	.0280	2.220	20.4	
Platinum	.0025	.442	22.1	21.2
	.0050	.917	22.9	
	.0075	1.284	21.4	
	.0100	1.661	20.7	
	.0125	1.913	19.1	
Tinfoil	.0049	.560	28.5	28.1
	.0098	1.157	29.6	
	.0147	1.681	28.5	
	.0196	2.040	26.0	
Water	.47	.028	.16	
	1.41	.109	.21	
	2.82	.240	.23	
Alcohol	.94	.042	.15	
	2.82	.153	.18	
Copper chloride	.47	.589	2.35	
	.94	1.134	2.27	
	1.41	1.673	2.48	
Lead nitrate	.235	1.060	8.8	
	.470	1.839	7.7	
Potassium iodide	.141	.987	11.2	
	.376	3.113	13.2	

TABLE III.—*Continued.*

Very Hard Tube, over 100,000 *Volts.*

Substance.	Thickness in Cms.	Equivalent of Al.	Relative Absorption.	
Carbon	1.27	.208	.27	
	2.54	.482	.33	
	3.81	.733	.34	
	5.08	.945	.33	
Sulphur	.625	.668	1.4	
Iron	.0165	.316	6.6	
	.033	.500	5.2	
	.0765	1.116	5.1	5.2
	.1305	1.641	4.4	
	.2175	2.882	4.6	
Nickel	.0495	.896	5.5	
	.0990	1.830	5.6	5.5
	.1485	2.610	5.4	
Copper	.045	1.03	6.9	
	.100	2.46	7.4	6.8
	.145	2.89	6.0	
Zinc	.0142	.372	9.6	
	.0287	.646	8.2	
	.0490	.954	7.1	8.0
	.0777	1.760	8.3	
	.1270	2.420	7.0	
Silver	.0040	.317	20.2	
	.0123	1.116	23.5	21.1
	.0246	2.046	21.4	
	.0369	2.790	19.4	
Platinum	.0025	.415	20.7	
	.0050	.750	18.7	
	.0060	.952	19.5	
	.0075	1.116	18.7	18.5
	.0120	1.860	19.4	
	.0170	2.252	16.5	
	.0195	2.600	16.3	
Tinfoil	.0049	.460	23.4	
	.0098	.930	23.7	22.3
	.0196	1.674	21.3	
	.0248	2.050	20.7	
Lead	.0052	.480	23.2	

A number of the results are shown plotted in Fig. 5, where the ordinates represent the thickness of layer and the abscissæ the equivalent thickness of aluminum. The curves are practically straight over a considerable range, with the exception of silver, whose anomalous behavior is indicated by the curvature, resembling a parabola. In the case of the very hard tube the curve for silver is nearly straight, showing that the ratio of the absorption by silver to that of aluminum is constant. This would

indicate that the rays examined were practically homogeneous. The ray before reaching the ionization chambers had to pass through about 6 sheets of ordinary tinfoil which would cut out any soft rays which might be present.

The tube ran in too irregular a manner to get any very accurate measurements by the rate of leak method, but a few approximate ones were obtained which gave for aluminum a value of λ/d = about .22. These hard X rays thus gave a value for aluminum only about four

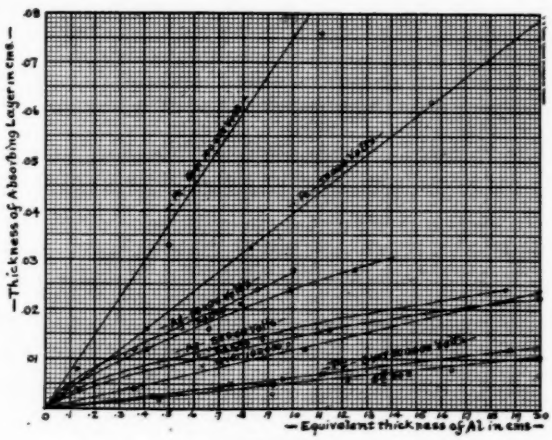


Fig. 5.

times greater than the γ rays did. For lead the value of λ/d for the X rays calculated from the relative absorption would be about 5.1, which is about twenty-five times the value of λ/d for the γ rays. The hardest X rays here obtained are still considerably softer than the softest γ rays, but it would seem that if the absorption by any material for the X and γ rays ever becomes equal it will be reached by the light atoms long before the heavy ones.

It can be seen that even for the hardest rays silver has a very high absorptive power.

CONCLUSION.

The chief points brought out in this paper can be conveniently summarized as follows:

1. For the soft γ rays λ/d follows the same general characteristics as already found for the hard rays, but increasing with the atomic weight in a more rapid manner.

2. For the X rays, λ/d, examined over a wide range and by different methods, gives results which are in general agreement with those of Benoist. λ/d is an increasing value of the atomic weight in general, but some substances, notably that of silver, show very anomalous results.

The value of λ/d for silver is much less for the soft rays, and much greater for the hard rays, than one would be led to expect.

3. Very hard X rays were obtained whose coefficient of absorption measured by aluminum was only four times greater than that of the γ rays. These rays are nearly homogeneous, and still show a very high absorbing power for silver.

4. None of the very high weight atoms, such as lead, platinum, etc., show any marked anomalous results. The attempt therefore to obtain X rays as hard as the softest γ rays has not been altogether successful, but it has been approached, and it has been shown that the anomalous results still persist for the X rays. Since the γ rays have not been shown to exhibit the same anomalous results, and if we assume that γ and X rays are the same in nature, a very rapid change in the absorptive powers of certain elements must take place as the X rays approach closely to the γ rays.

The anomalous absorption of silver has not yet been completely explained, but it is quite probable that the fluorescent secondary X radiation discovered by Barkla must have much to do with it. This radiation from silver is fairly hard and would not be excited by soft primary rays, but to some extent by medium hard rays, and largely by very hard rays. The fluorescent radiation could only have an effect when the primary beam was of sufficient hardness to excite strongly the radiation characteristic of the element under consideration. The heavy atoms should show no very great anomalous behavior until the very hard primary rays were used.

The latest results on the fluorescent radiation by Chapman[1] show that the high atomic weight elements give off a soft homogeneous radiation similar in character to that which Barkla calls series *L*. There is also, from these atoms, a considerable scattering of the primary, and some hard radiation of series *K*.

In a paper in the September number, 1911, of the Phil. Mag. Barkla gives a good summary of the present knowledge on the fluorescent radiation and its effect upon the absorption curves of any element for X rays. He ascribes all selective or anomalous absorption to the presence of the homogeneous radiation characteristic of the particular element under investigation. In the neighborhood of an absorption band, *i. e.*, at the point where the fluorescent radiation starts for any given element, the absorption, in terms of an element like aluminum having no absorption band, would increase rapidly at first and afterwards decrease. The characteristic form of an absorption curve would be the same for all elements. An absorption band would result from an abnormal absorp-

[1] J. C. Chapman, Proc. Roy. Soc., May, 1912.

tion of energy from the primary beam due to the sudden transformation into the fluorescent radiation.

A confirmation of this view of absorption is seen from the author's results for silver. Barka gives for the value of λ/d for the homogeneous radiation from silver in terms of aluminum, 2.5. From Table II. above we see that for a tube of about 13,000 volts P.D. the value of λ/d for aluminum is from 6.7 to 3.4, and therefore below that necessary (2.5) to start the homogeneous radiation. Under these conditions the relative absorption of silver in terms of aluminum is practically constant, and of low value. In the next tube running at 35,000 volts P.D. λ/d for Al = (.9 − .5), above that required. The relative absorption of silver in terms of aluminum is now much greater. Between these two hardnesses of tube the absorption band for silver occurred.

The results for the other elements experimented upon also show confirmation of this theory, but not to so marked a degree.

The relative absorption for most of the elements increases to a maximum and then decreases with the hardness of the rays, except silver, which over the range experimented upon has not begun to decrease appreciably.

From the results of Benoist and others it would seem that the elements round about silver exhibit the most marked anomalous characteristics.

If the X and γ rays are the same in nature but only differ in penetrating power, we might expect to find fluorescent radiations for γ rays. If this were true the absorption curves for γ rays would be affected in an analogous manner to X rays.

It is known that the secondary γ rays are of two kinds, high velocity β rays, and penetrating γ rays of the same nature as the primary. These latter rays according to Florance[1] are simply the scattered primary, of the same penetrating power, and not homogeneous.

There is no evidence yet of any homogeneous γ rays characteristic of the various elements.

In this respect then X rays and γ rays differ in character, and since it has been clearly shown that the hardest X rays excite fluorescent radiation, and that silver maintains its anomalous behavior, the γ rays if they are of the same nature as X rays must have lost this power rather suddenly.

The various theories which have been advanced to account for the production of X and γ rays do not seem complete enough at present to account for all the phenomena connected with γ and X rays.

University of Cincinnati,
September, 1912.

[1] Florance, Phil. Mag., 1910.

STUDY OF RESISTANCE OF CARBON CONTACTS.

BY A. L. CLARK.

WHILE engaged in some experimental work with telephone transmitters, it was found that the resistance of carbon contacts varied to a remarkable extent, not only with pressure and current, but also with the time and in a very regular manner.

The first two effects have been known for a long time, but the regular variation with time after eliminating the effect of change due to a rise of temperature, seems to have escaped notice—at least I have been unable to find any account of such work. To be sure it has been well known for a long time in telephone practice that oxidation and breaking down of contacts occur, but no attempts to show any regularity in such phenomena have been discovered.

The effects of pressure on contact resistance of carbon was investigated very thoroughly by Bidwell who also[1] investigated the effects of current strength on contact resistance. He found that the passage of a current through the contact caused the permanent change in resistance, sometimes decreasing it and sometimes increasing it, depending on the value of the current. In this connection, he found that if the current does not exceed a certain value, the resistance is decreased and more as the current is larger, provided the current does not rise above this value. On the other hand, if the current exceeds a certain value there is a permanent increase in the resistance.

At the time of the investigation described below, the writer did not have access to Bidwell's paper, consequently much of that work was repeated, but the regularity of the phenomena just described seems to be new and worth notice and such matters as are not mentioned in other researches are described below.

This paper is an account of an investigation of the time relations of changes in resistance in carbon contacts such as are commonly used in telephone transmitters. Experiments were made on carbon in the form of grains, balls and plates, always with the same results.

It will be convenient to divide carbon contacts into two classes, loose and tight. By a loose contact its mean one where the particles are held so that the contact is made under very slight pressure. In this case there

[1] Shelford Bidwell, "Elec. Res. of Carbon Contacts," Proc. Roy. Soc., XXXV., 1, 1883.

is no change of resistance due to expansion, which usually causes increased pressure, but the contact may break down, as will be seen below. By tight contact is meant one where the particles are held with sufficient firmness to maintain electrical contact even with large currents. In the first case, that is, loose contact, there are noticeable permanent changes in resistance. In the latter, tight contact, these changes are very slight. Microphonic action is not impossible in the case of tight contact. Indeed, the microphone is usually an example of tight contact in the sense explained above.

Loose contact was studied first by attaching a small carbon ball or a small grain of carbon to a thin phosphor bronze wire with a bit of a liquid cement, containing graphite and known as "clamp paste." This hardens in a few hours, is very strong and conducts well enough. In other experiments, the carbon balls and fragments were partially coated with copper electrolytically and then soldered to the phosphor bronze wire. Any other small wire possessing considerable elasticity would do equally well, but the phosphor bronze wire was at hand and served very nicely. The method of attaching the wire to the fragment seems to have no effect on the result. The wire was about 5 cm. long and was held by its end in a clamp so that the bit of carbon could be moved to touch another rigidly held piece of carbon. The circuit is shown in Fig. 1.

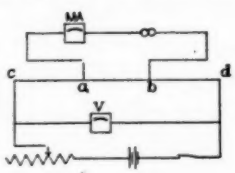


Fig. 1.

A 4-volt storage battery is arranged to send its current through a potentiometer wire, *cd*, and an adjustable resistance *R* in series. A sensitive voltmeter is applied at the ends of the wire, *cd*. The variable contacts *a* and *b* lead the current through a milammeter and the experimental carbon contact in series. The milammeter should be as near dead beat as possible. By moving *a* and *b* any required voltage may be applied and the current observed. It was found that when a small current is sent through the contact that there is a very regular decrease in resistance which if the contact is undisturbed seems to be permanent. The slightest jar, even that made by a wagon passing in the street outside, is sufficient to change the contact in such a way that the phenomenon repeats. Bidwell noticed that the resistance of such a contact is decreased by the current, but did not notice the regularity of the effect. Fig. 2 shows the manner in which the resistance changes. Here carbon balls ¼″ in diameter were used. The time was measured by a metronome beating 40 beats to the minute. If a stronger current is used, the decrease is more noticeable, and if a strong current is followed by a weak

one, the weak one has no effect on the resistance. It is not absolutely certain that there is no recovery from this change of resistance, but there is no experimental evidence to show that such a recovery occurs. Since there was no room available which could be made sound and jar proof, this point has not been carefully investigated. The sensitiveness of a loose contact to change in pressure is most extraordinary. When standing near the table on which the apparatus was placed, it was found that shifting the weight from one foot to the other distorted the floor and table enough to change the pressure sufficiently to show a decided effect in the milammeter.

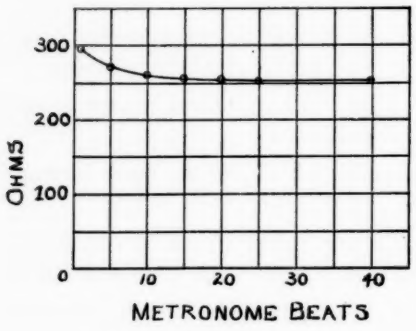


Fig. 2.

Table I. shows the result of several successive runs, beginning with a low E.M.F. and increasing to a value below the critical point. It will be seen that after the first run the resistance changes very little.

TABLE I.

E.M.F. in Volts.	I.	II.	III.	IV.
.4	445	308	296	296
.6	383	289	287	277
.8	254	222	236	236

If, as Bidwell points out, the E.M.F. passes a certain critical value, the phenomenon changes. There the resistance increases with the time and with remarkable regularity. As Bidwell noticed, the critical value depends somewhat on the pressure at contact. It also depends on contact area, so that the critical value probably depends on current density rather than on E.M.F. The higher the applied E.M.F., or better the higher the current density, the more rapid is the rise in resistance. Fig. 3 shows the phenomenon for two balls in loose contact when the applied E.M.F. is 1.6 volts.

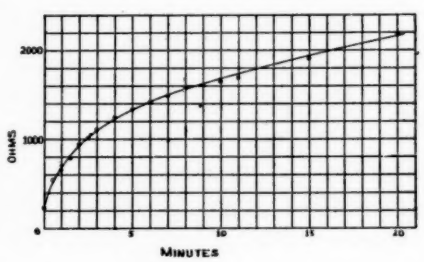


Fig. 3.

As will be seen, the resistance increases rapidly at first, then more slowly

and after about two minutes has increased to five-fold the original value. There is no evidence of recovery in this case. If the current be interrupted at any time and the circuit be left open for an hour or two and then closed, the resistance begins with the same value as when the current was interrupted and the curve is absolutely continuous, unless the contact has been jarred. With large E.M.F.'s but still so small that no visible arc forms, the resistance rises rapidly to infinity.

With smaller E.M.F.'s the change is very slow. If the initial value of the E.M.F. is near the critical value, it may happen that when the current is turned on the resistance falls for a moment, then rises. Apparently the fall in resistance gives rise to a current density in excess of the critical value and the second case is exhibited. Occasionally the resistance falls very quickly to a very low value and the contact then shows signs of cohesion, requiring a slight but unmistakable pull to separate the particles. Of course the contact is no longer loose. On the other hand, when the phenomenon of increase of resistance is observed and the particles are observed under a high power microscope, traces of a feathery adhering substance are sometimes seen, suggesting ash. This is not always visible and where seen may be small fragments abraded by rubbing, but the existence of ash is suggested.

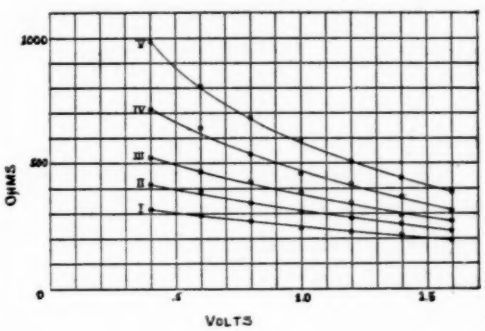


Fig. 4.

Table II. and Fig. 4 show the effect of successive passage of the E.M.F. beyond the critical value. Each such passage gives rise to an increase of resistance. In this experiment, the circuit was closed for as brief a period as possible. The longer the circuit is closed on the high value of the E.M.F. the higher the next curve of the family will rise.

TABLE II.

E.M.F.	I.	II.	III.	IV.	V.
.4	313	413	523	712	988
.6	290	382	462	635	808
.8	267	342	425	533	684
1.0	243	307	380	463	592
1.2	223	282	340	411	504
1.4	212	258	294	362	445
1.6	186	229	274	309	385

The phenomena observed are totally different when the pressure becomes great enough to break through the nonconducting layer, whatever it is. For example, a ball was held against a vertical disk of carbon with pressure produced by the elasticity of the phosphor bronze wire and the phenomena of loose contact were observed. It was then placed on the same disk held horizontally, the pressure now being caused by the weight of the ball and the contact was tight. When the contact is tight there is no breakdown, even when the current is large enough to cause a bright arc. In another experiment, two thick carbon disks were placed near together in the same vertical plane and a small fragment of carbon dropped between them. Such a contact may be loose or tight, behaving very erratically.

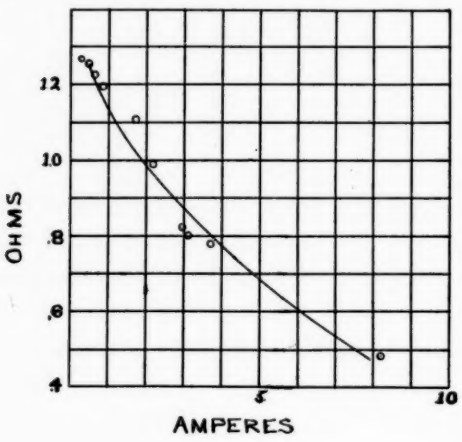


Fig. 5.

Fig. 5 shows the change in resistance due to different currents in the case of tight contact. In this experiment, a small ball was held against a carbon disk with moderate pressure. As will be seen from the readings in Table III., it is doubtful whether there is a decided change in resistance with time. The first column shows time in half minutes. The other columns show the resistances at these times for the applied electromotive forces given at the top of the columns.

TABLE III.

Time.	.06 Volt.	.21 Volt.	.4 Volt.	.56 Volt.	.72 Volt.	.83 Volt.	1.09 Volts.	1.60 Volts.	1.9 Volts.
0	4	3	2.9	3.4	3.2	2.82	2.66	1.54	1.29
½	4.8	3	2.9	3.38	3.18	2.77	2.68	1.55	1.23
1	4.4	3	3.2	3.3	3.25	2.99	2.6	1.54	1.23
1½	4.2	3	3.3	3.28	3.25	2.88	2.6	1.50	1.25
2	4.4	3	3.28	3.33	3.22	3.03	2.55		
2½			3.27	3.34	3.22	2.92			
3			3.26	3.34					

In some cases of tight contact, *e. g.*, a small ball against a disk under slight pressure, it has been found that the resistance decreases with time, but if the current be interrupted by opening the switch for a few minutes and closing it again, the resistance has a much smaller value than when

the switch was opened. The following data will make this evident. With an E.M.F. of four volts the resistance fell in four minutes from 158 to 146 ohms. After a rest of two minutes the resistance was 135 ohms and the circuit being closed for three minutes, no further change in resistance was observed. But a rest of two minutes brought the resistance down to 99 ohms, where it remained practically constant for three minutes. Another two-minute rest brought it to 91 ohms. In experiments like this each period of rest brought about a marked decrease in resistance. This phenomenon has been observed a number of times, but no satisfactory explanation has been suggested.

QUEEN'S UNIVERSITY,
KINGSTON, ONTARIO.

ANGLE IN VECTOR ALGEBRA; AND COMPOSITION OF ROTATIONS.

BY FREDERICK SLATE.

ALTHOUGH angular displacement, velocity and acceleration have their place as axial vectors recognized in vector algebra, the foundations of their acceptance into the scheme are seldom laid in satisfactory fashion, and their relation to the kinematics of a rigid body is not always stated accurately. The ideas here offered have for object to supplement the view of these points that is usually presented, and so render it systematically more complete.

The specification of plane angle as the quotient of circular arc by radius includes no orienting elements explicitly, and is consequently inadequate for the purposes of vector expression. But the orientation can be supplied easily and without affecting the ordinary measure of magnitude. Writing for differential angle

$$d\mathbf{a} = \frac{ds}{r}[\mathbf{r}_1\mathbf{t}_1] \tag{1}$$

the appended vector product determines the plane of the angle-element by means of unit vectors in the radius and the tangent for the element of circular arc (ds), while the tensor assigns proper magnitude (ds/r). For this primary form we can then substitute the convenient equivalent

$$d\mathbf{a} = \frac{\mathbf{I}}{r^2}[\mathbf{r}d\mathbf{s}], \tag{2}$$

which clearly remains valid also for any radius vector ($\mathbf{r}$) and any curve-element ($d\mathbf{s}$) at its extremity. This original limitation to differentials becomes necessary in order that the unit vectors may be specified locally; but it disappears effectively when one plane contains the pole and a finite arc. In that simpler case

$$\mathbf{a} = \int_0^{s_1} \frac{\mathbf{I}}{r^2}[\mathbf{r}d\mathbf{s}] \tag{3}$$

is evidently to be taken as an algebraic summation of coplanar areal elements representable on a common normal, sign in the individual contributions to the total being rendered consistent with the standard con-

ventions by the sequence chosen for the factors in the vector product. Although the vector factors separately vary from one element to another, their product is in a correct sense common to all the terms summed when eq. (1) or eq. (2) is integrated. The more general conditions, however, are seen to involve the interpretation of the process in eq. (3) as a vector-summation.

Plane angle or angular displacement is thus fully established as an axial vector through this necessary connection with the "area described by the radius vector"; being represented as twice the area of the corresponding sector in a circle of unit radius. And without need of further discussion it is then apparent how plane angles, finite as well as infinitesimal, must conform without exception to the rules governing other axial vectors. And further this entails logically a similar conclusion regarding angular velocity and angular acceleration through their aspect as increments of angle, dimensional differences among vectors of the same type being reconciled, here and elsewhere, by familiar considerations.

Turning now to solid angle, begin by assuming a surface-element (dS) on a sphere of radius (r); next extend the scope of the symbols to the case where an element of any surface is taken at the extremity of its radius vector; and notice that all accepted details in the measurement of solid angle (A) fit naturally into the differential expressions

$$dA = \frac{dS}{r^2}(\mathbf{r}_1\mathbf{T}_1) = \frac{\mathbf{I}}{r^3}(\mathbf{r}d\mathbf{S}). \tag{4}$$

These have been so constructed as to introduce a scalar product; first of unit vectors orienting the radius vector ($\mathbf{r}_1$) and the surface-element ($\mathbf{T}_1$), and secondly of vectors parallel to these, but including also the tensor factors. The operations indicated leave the measure of magnitude undisturbed. And the result being scalar the need of discrimination that attaches to eq. (3) does not enter at the next stage; so that we have in general the scalar sum

$$A = \int_0^{S_1} \frac{\mathbf{I}}{r^3}(\mathbf{r}d\mathbf{S}), \tag{5}$$

which is interpretable immediately as a volume three times that of the spherical sector of unit radius which ordinarily appears in the discussion of solid angle. The correct signs among the elements constituting the algebraic sum follow automatically from the individual cosine-factor which is implied in each scalar product ($\mathbf{r}d\mathbf{S}$).

The application to the kinematics of a rigid solid turns upon the expression of linear displacement at any definite point in terms of the

"angular displacement of the solid." There are two current plans of calculation, according as the origin is taken (1) at the center of mass (which may then itself be moving), or (2) at a fixed point in unchanging configuration with the solid. We shall make the development definite by choosing the latter alternative: there is no essential divergence of results, for the purpose here. From the origin (O) draw a fixed line (OA) about which angular displacement (a_1) is supposed. Then for any selected point (Q) the radius vector (r) must be of constant length, but is subject to change (in direction) as a vector; and for differential linear displacement

$$d\mathbf{r} = d\mathbf{s} = [d\mathbf{a}_1\mathbf{r}]. \tag{6}$$

Combination with eq. (2) shows the necessary identity, $d\mathbf{s} = d\mathbf{s}$. Proceeding to finite angular displacement, linear displacement is shown as a vector sum (though the elements of angle are additive algebraically), of obvious geometrical meaning,

$$\int_{r_0}^{r_1} d\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_0 = \int_0^{a_1} [d\mathbf{a}_1\mathbf{r}]. \tag{7}$$

After completing this displacement let further angular displacement occur about any other fixed line (OB). Then similarly

$$\mathbf{r}_2 - \mathbf{r}_1 = \int_0^{a_2} [d\mathbf{a}_2\mathbf{r}], \tag{8}$$

and by addition preserving the sequence of integrals,

$$\mathbf{r}_2 - \mathbf{r}_0 = \int_0^{a_1} [d\mathbf{a}_1\mathbf{r}] + \int_0^{a_2} [d\mathbf{a}_2\mathbf{r}]. \tag{9}$$

The second member does not coalesce into the form

$$\int_0^{a} [d\mathbf{a}_1 + d\mathbf{a}_2, \mathbf{r}]$$

for the plain reason that the radius vector is not a common factor (as a vector) in pairs of elements that can be segregated one from each integral; and moreover the series of its (vector) values in either integral would not be reproduced if the sequence were reversed. We can detect in these facts (1) the foundation of Rodrigues' Theorem, and (2) the harmony of that theorem with full exhibition of vector qualities in angle, as set forth in what precedes. For note that the necessary and sufficient condition rendering legitimate and equivalent the combinations

$$\mathbf{r}_2 - \mathbf{r}_0 = \int_0^{a} [d\mathbf{a}_1 + d\mathbf{a}_2, \mathbf{r}] = \int_0^{a} [d\mathbf{a}_2 + d\mathbf{a}_1, \mathbf{r}] = \int_0^{a} [d\mathbf{a}\mathbf{r}] \tag{10}$$

is that the vector denoted by the symbol (**r**) should be made common (or unique); and this is met by supposing the processes of displacement simultaneous. The limitation to infinitesimal angle is secondary then, as an accompaniment of simultaneousness; and at most marks a progressive temporary (or instantaneous) adjustment within an interval of finite displacement.[1] The remaining steps in adapting the thought of eq. (10) to angular velocities and accelerations are so evident that they need not detain us, except to emphasize the simultaneousness again as dominant, for instance in the "Parallelogram of Angular Velocities," thus: Consider rotation ($\dot{\mathbf{a}}_x$) about the X-axis as occurring *alone*, and producing linear velocity ($\mathbf{v}_1$) at a point given by its distance ($\mathbf{r}_1$) from that axis; and similarly for rotation about Y-axis and Z-axis, each alone. Then

$$\mathbf{v}_1 = [\dot{\mathbf{a}}_x \mathbf{r}_1]; \quad \mathbf{v}_2 = [\dot{\mathbf{a}}_y \mathbf{r}_2]; \quad \mathbf{v}_3 = [\dot{\mathbf{a}}_z \mathbf{r}_3]. \tag{11}$$

It is next required to adjust the *simultaneous* occurrence of the velocities in eq. (11) to producing the actual motion. That is, introducing the radius vector, to satisfy the condition

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = [\dot{\mathbf{a}}_x, \mathbf{r} - \mathbf{x}] + [\dot{\mathbf{a}}_y, \mathbf{r} - \mathbf{y}] + [\dot{\mathbf{a}}_z, \mathbf{r} - \mathbf{z}]. \tag{12}$$

But $[\dot{\mathbf{a}}_x \mathbf{x}] = 0$, etc.; so we may write

$$\mathbf{v} = [\dot{\mathbf{a}} \mathbf{r}]; \quad \text{where } \dot{\mathbf{a}} = \dot{\mathbf{a}}_x + \dot{\mathbf{a}}_y + \dot{\mathbf{a}}_z; \tag{13}$$

and find the accurate interpretation of the scope and meaning of the result in equations (6) to (10).

UNIVERSITY OF CALIFORNIA.

[1] These details may be justified by Heaviside's remark that "This property is notoriously difficult to understand by Cartesian mathematics"; and we may entertain a doubt whether his vector proof is not over-simplified (El. Mag. Theory, Vol. 1, p. 181). The older books sometimes were content with showing that the resultant axis is a line of zero velocity. Denial of vector quality to angle is a modern confusion (Coffin, Vector Analysis, 2d ed., p. 241).

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SIXTY-FOURTH MEETING.

A REGULAR meeting of the Physical Society was held in the Physical Laboraory of Northwestern University, Evanston, Ills., on Saturday, November 30, 1912.

In the absence of the president and vice-president Mr. Crew was made chairman of the meeting.

The secretary made a report to the meeting regarding the mail ballot held in May, 1912, the report being in substance the same as that presented at the New York meeting on October 12, 1912, and given in the minutes of that meeting.[1]

At the close of the meeting the Society voted to express to the authorities of Northwestern University, to the members of the Department of Physics, and to the University Club, its high appreciation of the hospitality extended to the Society.

The following papers were presented:

Sound Wave Photography. ARTHUR L. FOLEY.

A Method of Producing Known Relative Sound Intensities and a Test of the Rayleigh Disc. G. W. STEWART and HAROLD STILES.

The Effects of Gas Currents on Sound, with Applications to Ventilating Systems of Auditoriums. F. R. WATSON.

The Value of e. R. A. MILLIKAN.

The Cooling Effect Produced by the Emission of Ions from Hot Bodies. H. L. COOKE and O. W. RICHARDSON.

The Evidence that Sodium belongs to a Radioactive Series of Elements. F. C. BROWN.

Ionization of Potassium Vapor by Ultraviolet Light. S. HERBERT ANDERSON.

Photographs of Retrograde Rays. CHAS. T. KNIPP.

Note on the Distribution of Velocities of Photo-electrons from Thin Cathode Films when Illuminated by the Mercury Arc. PAUL H. DIKE and F. C. BROWN.

New and Old Expressions for the Variation of the Mass of the Electron with Increasing Velocity. JAKOB KUNZ.

[1] PHYSICAL REVIEW, XXXV., p. 397.

The Relation between Current and Time Rate of Change of Electric Force. MAX MASON.

The Stability of Residual Magnetism. N. H. WILLIAMS.

Summary of a Study of the Electric Furnace Spectrum of Iron. (By title.) ARTHUR S. KING.

The Infra-red Arc Spectrum of Iron in the Region 8000–30000 A.U. H. M. RANDALL.

Distribution of the Elements in the Solar Atmosphere. CHARLES E. ST. JOHN.

Some Direct Measurements of the Temperature Correction to the Refractive Index of Water around 20° C. F. A. MOLBY.

The Effect upon the Elastic Properties of Copper Wire Produced by Increase of Temperature and Passage of an Electric Current. H. L. DODGE.

A Method of Production of Light-negative Selenium. F. C. BROWN.

A Substitute for a Bronson Resistance. (By title.) D. W. CORNELIUS and J. G. KEMP.

Air Resistance to Falling Bodies. (By title.) A. A. SOMERVILLE.

The Effect of Drawing on the Elasticity of Certain Wires. L. P. SIEG.

On the Cause of the Apparent Difference between Spark and Arc Sources in Imparting Initial Speeds to Photo-electrons. R. A. MILLIKAN.

Note on a Possible Kinematic Explanation of Spectral Series with Constant Frequency Intervals. ERNEST MERRITT.

Adjourned at 5 P.M.

ERNEST MERRITT,
Secretary.

MINUTES OF THE SIXTY-FIFTH MEETING.

A JOINT meeting of the Physical Society with Section B of the American Association for the Advancement of Science was held in the physical laboratory of the Case School for Applied Science, Cleveland, Ohio, December 30, 1912, to January 1, 1913. The presiding officers were President W. F. Magie, of the Physical Society, and Vice-President A. G. Webster, of Section B.

The annual business meeting of the Physical Society was held Wednesday afternoon, January 1, 1913.

The chair appointed Messrs. F. R. Watson and F. A. Saunders to act as tellers for the annual election of officers and, the tellers having reported the result of their count of the ballots, the following officers were declared elected for the year 1913:

President: B. O. Peirce.
Vice-President: Ernest Merritt.
Secretary: A. D. Cole.
Treasurer: J. S. Ames.
Members of the Council: H. T. Barnes and W. J. Humphreys.

On behalf of the Council the secretary announced that the next meeting of

the society, which would ordinarily be held in New York City on the Saturday nearest March 1, would be held in the new Sloane Physical Laboratory of Yale University at New Haven, the exact date to be announced later.

The secretary also announced that the Council had voted to continue the present arrangement with the publishers of Science Abstracts for the year 1913.

The meeting voted to approve the action of the Council in ruling that papers which contain chiefly matter that has already been published be not accepted for presentation before the Society.

In the absence of Mr. Ames, the treasurer's report for the year ending December 1, 1912, was read by the secretary. The report is printed at the close of these minutes.

The president announced to the society the action of the council at its meeting December 31, 1912, in regard to the proposed transfer of the Physical Review to the Physical Society, and gave an account of the steps which had led to this action. Owing to the indecisive result of the mail ballot of May, 1912, the editors of the Review had on their own account sent out a circular letter to all regular and associate members, a copy of which letter follows:

"*To the Members of the American Physical Society:*

"The question of the advisability of transferring the control of the Physical Review, and the responsibility for its maintenance, to the Physical Society has now been under consideration by the editors of the Review and the Council of the Society for nearly two years. In March, 1911, the editors made a statement to the Council in regards to the terms of such a transfer (page 593 of the Review for June, 1911) and in February, 1912, made a further statement looking toward a transfer of the Review to the Society on January first, 1913. There are reasons why, if the transfer is to be made, it should be made not later than that date.

"It will be recalled that in May, 1912, an effort was made by the Council to ascertain the sentiment of the Society by means of an informal mail ballot, in which all regular members were asked to participate. Of the 201 votes cast, 97 were in favor of the transfer; 87 were in favor of leaving the relations between the Society and the Review unchanged; and 16 were in favor of the establishment by the Society of a new journal. 115 members did not vote.

"Since less than a majority of the regular members (in fact less than one third) expressed themselves as in favor of the change, one of the essential conditions for the transfer has not been met, and technically the vote must be interpreted as indicating that the editors' offer has been declined. The ballot is, however, not convincing; for while it fails to give evidence of such enthusiastic approval of the plan as has been felt by the editors to be essential to its success, it nevertheless indicates a considerable sentiment in favor of the change.

"Through correspondence and discussion with individual members of the Society, the editors of the Review have been led to believe that the failure of the ballot to give convincing expression to the sentiment of the Society may

have been due in part to a failure to understand the editors' attitude toward the proposed change. It is possible that our attempt to maintain an impartial attitude in the discussion has been interpreted as indicating that we are opposed to the plan. We are sending the following statement of our views to all members of the Society in the effort to make our position more clear, and in the hope of obtaining an expression of opinion from the members of the Society which we are certain is not based upon a misunderstanding.

"At the time when the transfer of the Review to the Society was first discussed, the expense of publication exceeded the income by a considerable amount; for, although the income was steadily increasing, the amount of material published had increased more rapidly. During the last two years, however, the situation has materially changed. With the effective coöperation of the advisory editors it has been possible, by urging a more compact style of presentation, to secure a considerable reduction in the bulk of material published. This fact, together with the various economies that have been introduced, has resulted in a decrease in the cost of publication. On the other hand the income has shown a very gratifying increase, with the result that during each of the last two years the income has exceeded the expense of publication.

"If the Society should undertake the maintenance of the Review the expense to the Society would be greater than it now appears to be by the amount of the editor's salary. But even taking this into consideration we believe that if the Review is managed in a conservative way, and if the amount of material published is not greatly increased, the transfer may not make necessary either the discontinuance of Science Abstracts or any great increase in the dues. No immediate increase in dues appears to be necessary. It must not be forgotten, however, that if the rapid increase in scientific activity continues, or if the transfer to the Society should result in the publication in the Review of the American material which now goes elsewhere—and this result is certainly to be desired—the expense of publication would be correspondingly increased. It might be that the very success of the change would make some increase in dues necessary. In considering the advisability of making the transfer, this possibility should not be left out of account. If the transfer is made the members should feel that they are taking the Review 'for better, for worse.'

"The conditions under which the editors are willing to make the transfer to the Society have been stated in our earlier communications, these conditions being made for the purpose of safeguarding not only the interests of the journal but also the interests of the members of the Society. The establishment of a scientific journal is a slow process. Even under the most favorable conditions, aside from the expenditure of money and effort, it requires a long time to build up a subscription list, to obtain the patronage of advertisers, and to acquire general scientific recognition. We feel that the Review is now so firmly established that its continued success is assured. On behalf of Cornell University we are offering to the Society a journal with an established position,

with a subscription list that compares favorably with that of other similar journals, and with an annual income—in addition to that received from the Physical Society—of over $3,000. We feel that under the control of the Society, the Review can be made still more successful. But we cannot run the risk of losing what has already been gained. The conditions stated in our earlier communications appear to us to be essential; and of these the assured and hearty support of the members is the most important. Unless we are convinced that the members of the Society are prepared to take a vital interest in the success of the Review, and to insist that its policy and management are such as the physicists of the country desire, we feel that the surrender by the present editors of their control and responsibility would be little short of a breach of trust.

"If conducted by the Physical Society with the enthusiastic support of the members, we believe that the Review will be more successful, and more truly representative of American Physics, than it could possibly be under private ownership or when controlled by a single institution as at present. On the other hand, if the support of the members is lukewarm, we feel that lack of interest and divided responsibility might bring disaster upon the Review, and even upon the Society, and that, at the present time at least, the proposed transfer should not be made. The enthusiastic support of the members being assumed, we are heartily in favor of the transfer of the Review to the Society, and look upon this transfer as being to the best interests of both the Society and the Review.

"With this explanation of the situation and of the editors' views, may we ask that you indicate your preference on the enclosed postal card and return the card to us at once?

"If it appears that the transfer of the Review is strongly favored by the members, we intend to ask the advisory editors to assist us in preparing a definite plan for the control and management of the Review as the journal of the Society, and shall present this plan for consideration at the Cleveland meeting. If the members do not feel that the change is desirable we shall proceed to perfect and make more definite the present plan of a representative editorial board to coöperate with the present editors.

"We cannot too strongly urge that you mark and return the enclosed card at once whatever your opinion on the question may be.

"December 11, 1912." "EDW. L. NICHOLS
ERNEST MERRITT
FREDERICK BEDELL."

The replies to this letter had shown so large a majority in favor of the transfer of the Review to the society that the editors felt that they were justified in approving the transfer and had therefore presented to the council the following proposition:

The Physical Society takes over the control of the Physical Review and

assumes full financial responsibility for its maintenance under the terms proposed by the editors in their communications of March, 1911, and February, 1912 (printed in the Review June, 1911, and November, 1912).

The transfer shall be effective as of January 1, 1913, at which date a new series of the Review shall be begun, the January, 1913, issue being designated as "No. 1, Vol. I., Second Series."

The actual transfer of management shall be made as soon thereafter as the society can be incorporated and various necessary details can be arranged. This transfer shall not include the transfer of any tangible property, as office furniture, back volumes, etc., which property shall remain the property of the present management.

The complete control of the Review, including its business and editorial management, shall be vested by the Society in an editorial board elected by the Society.

This board shall consist of a managing editor and nine other members. The term of office shall be three years, and, except in the case of the managing editor, retiring members shall be ineligible for immediate reëlection. The council shall determine the salary of the managing editor and shall appoint a managing editor for the year 1913. The managing editor for 1914–16 and for subsequent terms shall be elected by the society.

The other nine members of the initial board shall be the present nine advisory editors, arranged in three groups as follows:

(*a*) B. O. Peirce, J. S. Ames, J. Zeleny;

(*b*) E. F. Nichols, J. C. McLennan, R. A. Millikan;

(*c*) W. F. Magie, K. E. Guthe, C. A. Skinner.

One group (determined[1] by lot) shall retire at the end of 1913; a second group (determined by lot) shall retire at the end of 1914; the third group shall retire at the end of 1915.

The board shall receive all money paid to the Review for subscriptions, sales, etc., shall receive from the treasurer of the society remittances of such sums as the Society may appropriate, shall make all disbursements for the maintenance of the Review, and shall make a report to the council each year of such receipts and disbursements. The treasurer shall place at the disposal of the board of editors one thousand dollars working capital and the sum of three dollars per year for each member of the Society until the council otherwise directs.

After extended discussion the council had voted to accept the proposition and to adopt the plan proposed by the editors of the Review subject to the approval of the meeting.

It was further announced that the council had voted, in case the action proposed above was approved by the meeting, to appoint F. Bedell managing editor of the Review for the year 1913.

[1] The Board of Editors subsequently determined by lot that group *a* should retire at the end of 1913, group *c* at the end of 1914, and group *b* at the end of 1915.

It was moved and seconded that the meeting approve the action of the council, and after discussion by Messrs. Foley, Bedell, Carman, A. Zeleny, Guthe, Webster, Millikan, E. F. Nichols, McLennan, G. W. Stewart, Merritt, and others, the motion was adopted with no dissenting votes.

It was moved by Mr. Bedell that the managing editor be required to give bond for such amount as might be determined by the Board of Editors, the cost of such bond to be paid from the money appropriated to the Review, and that auditors be appointed annually by the president to audit the accounts of the managing editor. The motion was carried.

It was voted to place in the Review,—if possible in the first number of the New Series,—a statement in regard to the transfer of the Review to the Society, together with some account of the organization and history of the Review, and an expression of the society's appreciation of the work of the original editors. Mr. A. G. Webster was requested to prepare such a statement.

On motion of Mr. Webster the meeting voted to express to Mr. Merritt its appreciation of his services to the society as secretary during the past thirteen years.

The secretary called attention to the fact that the newly elected secretary, Mr. Cole, would be in Europe for several months and that the duties of the secretary's office could not be taken over by Mr. Cole until his return.

Tuesday afternoon, December 31, 1912, the Physical Society and Section B of the American Association for the Advancement of Science met in joint session with Section A of the Association. The program of the joint session was as follows:

Address of the retiring Vice-President of Section A, MR. E. B. FROST, on "The Spectroscopic Determination of Stellar Velocities."

Address of the retiring Vice-President of Section B, MR. R. A. MILLIKAN, on "Unitary Theories in Physics."

Henri Poincaré as a Mathematic Physicist. A. G. WEBSTER.

Some General Aspects of Modern Geometry. E. J. WILCZYNSKI.

Cosmical Magnetic Fields. L. A. BAUER.

The session of Wednesday morning, January 1, 1913, which was in charge of Section B was devoted to papers of general interest as follows:

Photographing and Analyzing Sound Waves. DAYTON C. MILLER.

The Reaction of the Room on the Source of Sound. WALLACE C. SABINE.

Some Points Concerning Absolute Measurements of Sound. ARTHUR G. WEBSTER.

In addition the following papers were presented during the meeting:

Notes on Problems Involved in Producing Three-color Transparencies. CHARLES D. HODGMAN.

Thermal Capacity of Carbon and Tungsten at Glowing Temperatures. A. G. WORTHING.

Resistance of Oxides at High Temperatures. A. A. SOMERVILLE.

A Micro-Pyrometer. GEORGE K. BURGESS.

Trial of a Method of Obtaining Equality of Temperature, at High Temperatures. C. F. LORENZ.

A Critique of Captain Kater's paper of 1818 on The Reversible Pendulum. JOHN C. SHEDD and JAMES A. BIRCHBY.

Some Diffraction Phenomena. C. F. BRUSH.

Effect of a Constriction in the Path of a Luminous Discharge. R. F. EARHART.

Arc and Spark Phenomena. E. S. JOHONNOTT.

The Effect of Magnetization on Thermal Conductivity. N. F. SMITH.

On the Origin of the Earth's Magnetic Field. L. A. BAUER.

A Study of the Joule and Wiedemann Effects in Nickel Rods. S. R. WILLIAMS.

On the Principle of Relativity. A. MACFARLANE.

A Demonstration of the Relativity Concepts of Time and Space by Means of a Model. REINHARD A. WETZEL. (By title.)

Note on a High Tension Oscillatory Ignition System. M. E. GRABER.

Reversal of the Rowland Effect. F. E. NIPHER.

Application of Quaternions to Curvilinear Coördinates. L. B. TUCKERMAN, JR.

Notes on the Hall Effect. ALPHEUS W. SMITH.

The Series System in the Spectra of Ca, Sr, and Ba. F. A. SAUNDERS.

Optimum Wave-length in Radio Telegraphy. A. H. TAYLOR.

Propagation of Signals in a Dispersive Medium. WALTER COLBY.

Absorption of β-rays by Air and Carbon Dioxide. ALOIS F. KOVARIK.

Note on Some Anomalies of the Kerr Magneto-optic Effect. L. R. INGERSOLL.

Standards of Wave-Length and Desirable Data for Them and for Other Lines. CHARLES E. ST. JOHN.

On Special Conditions in the Electric Furnace Which Produce a Spectrum Similar to that of the Spark. ARTHUR S. KING.

Resonance in the High Frequency Oscillations of a Secondary Circuit Excited by the Lepel Arc. A. D. COLE.

The Similarity of Electrical Properties in Light-positive Selenium to Those in Certain Crystal Contacts. F. C. BROWN.

The Effect of Increase of Temperature and Passage of an Electric Current upon the Elastic Properties of Soft Steel Wire. H. L. DODGE.

Vacuum Tube Discharge in a Magnetic Field. NORTON A. KENT and ROYAL M. FRYE.

Change in Electrical Conductivity Due to the Orientation of Oblate Spheroids Within the Conductor. S. R. WILLIAMS.

Spectra of Low Potential Discharges in Air and Hydrogen. (By title.) GORDON S. FULCHER.

The Brownian Movements in Gases at Low Pressures. R. A. MILLIKAN.

The Free Vibrations of a Lecher System using a Lecher Oscillator. F. C. BLAKE and CHAS. SHEARD.

The Velocity of Electrons in the Photo-electric Effect. D. W. CORNELIUS.

On Magnetic Rays. L. T. MORE.

Certain Minimum Insulating Distances between Conducting Surfaces, the Method of Measurement, and Reference to Electronic Atmospheres. F. C. BROWN.

The Earth Inductor as an Inclinometer. N. E. DORSEY.

A Method for Charging the Needle of a Quadrant Electrometer. HENRY A. ERIKSON.

Line Structure and Shift. NORTON A. KENT.

Phase Change by Reflection. C. A. SKINNER and L. B. TUCKERMAN, JR.

Some New Diffraction Photographs. MASON E. HUFFORD.

The Constants of Spectral Radiation of a Uniformly Heated Enclosure or So-called Black Body. W. W. COBLENTZ.

Some Observations on the Electric Spark in Air. ARTHUR L. FOLEY.

The Fluorescence Spectra of the Uranyl Salts at Low Temperatures. EDW. L. NICHOLS and ERNEST MERRITT.

The Difference of Phase at the Ears Produced by a Source of Sound not Distant. G. W. STEWART.

Physical Characteristics of Standard Vowel Tones. DAYTON C. MILLER.

The Use of Sounding Boards in an Auditorium. F. R. WATSON.

ERNEST MERRITT,
Secretary.

TREASURER'S REPORT FOR THE YEAR 1912.

Disbursements.

Executive expenses:	
President, mileage	$ 78.63
Secretary, mileage	121.86
Secretary, office expenses	100.00
Treasurer, office expenses	50.00
Stationery, printing and postage:	
Orders of secretary	248.95
Orders of treasurer	42.00
Subscriptions to Journals:	
Physical Review, 602 members	1,806.00
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Summary.

In bank December 1, 1911	$1,017.17
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J. S. AMES,
Treasurer.

I have verified the above accounts, examined the bank-balance, and certify that they are correct.

A. H. PFUND.

December 23, 1912.

THE INTRINSIC BRILLIANCY OF THE GLOWWORM.[1]

BY HERBERT E. IVES AND C. W. JORDAN.

PREVIOUS work on the firefly has shown its radiant luminous efficiency to be very high. All its radiation is in the visible spectrum, and localized in the yellow-green portion.

A point which has not been investigated, to the authors' knowledge, is the intrinsic brightness of the light. This is a matter of far greater importance than the quantity of light emitted, for the latter is dependent in a large measure on the insect's size and not necessarily on the peculiar process of light production. If the chemical laboratory of the firefly could be copied, would the product be of use to man? This depends upon the intrinsic brightness, for that determines the size of light necessary for any lighting requirement.

A species of glowworm (*Photuris Pennsylvanica*) accidentally observed in Philadelphia had the great virtue, from the standpoint of study, of giving a steady glow from its luminous abdominal areas. The spectral character of the light appeared to be the same as that previously observed in the firefly. The brightness of these spots was compared directly with that of a white blotting paper illuminated by a 10-candle-power standard incandescent lamp. By proper manipulation of the glowworms their bright areas were visible directly against the white surface, with no dividing line, making possible a very close match by moving the incandescent lamp to and fro.

It was found that the luminous spots were as bright as the blotting paper under an illumination of 190 meter candles, or considerably brighter than white

[1] Abstract of a paper presented at the New York meeting of the Physical Society, Oct. 12, 1912.

surfaces are ordinarily illuminated by artificial light (circa 25 meter candles). Taking the albedo of blotting paper as .76, it follows that the candle-power per cm² of the glow worm is .0046. In order, therefore, to obtain 25 M.C. on a plane 2 meters below a light source, the latter would have to be about 2 square meters in area. The source would give 100 c.p. which should be sufficient to illuminate a room of five times the area just given.

A peculiarity of most instances of chemi-luminescence is their excessively low specific brightness; on the other hand most artificial illuminants are of such excessive brightness that the light must be spread over greater area by the use of diffusing glass or reflecting surfaces. The intrinsic brilliancy of the glowworm lies between these extremes. It appears, therefore, that the solution, not only of the problem of light efficiency, but of light intensity, may follow from copying the process exhibited by the firefly.

THE EFFECT OF DRAWING ON THE ELASTICITY OF CERTAIN WIRES.[1]

BY L. P. SIEG.

IN a recent paper[2] the writer has pointed out that the elastic constants of certain wires were modified to a remarkable extent by drawing. This work has been continued on six wires specially drawn for the purpose by Heræus.

It was specified that the wires should be from the same alloy, a 40 per cent. alloy of iridium with platinum. As far as possible the wires were to be treated in exactly the same manner during the process of drawing. Six wires were thus prepared, ranging in diameter from 0.20 mm. to 0.10 mm. It was hoped from results of previous work that there would be a progressive change in the torsional constants with the drawing, but while marked changes have been found, they have not been found to be uniform, in spite of the fact that careful efforts were made to have the conditions exactly the same in the six experiments.

Just one aspect of the matter will be presented. The wires were annealed under constant load to a yellow heat, care being exerted to get this annealing temperature nearly the same for all the wires. The supported weight could be adjusted so as to change its moment of inertia without changing the load. The periods in the six cases were adjusted so that the effects of air friction were the same. The following table gives the results of only one aspect of the work. In a way these results represent the relative amounts of internal

Diameter (mm.).	Vibrations.
0.250	65
0.200	186
0.165	171
0.130	98
0.115	100
0.100	56

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] PHYS. REV., XXXV., Nov., 1912.

friction in the six cases. In the first column is the diameter of the wire used, while in the second is the number of vibrations executed by the wire in question as it died down from an amplitude of 18° to an amplitude of 5° per cm. of length of the wire.

There is an indication from the above that with the successive drawings there is formed a wire with gradually increasing rigidity, although the results from the first wire are disconcerting. It is strange to find that even the heating of these wires to a bright yellow heat is not sufficient to bring them to the same elastic condition, even though their chemical composition is exactly the same. Results, which will not be given here, on the relation of period of these pendulums with their amplitudes tend to emphasize the above peculiarities. This may account for the wide variation of elastic constants of common wires as obtained by different investigators, whose attention has been fixed perhaps on the chemical constitution of their wires more than on the methods used in the drawing.

STATE UNIVERSITY OF IOWA,
November 26, 1912.

THE COOLING EFFECT WHICH ACCOMPANIES THE EMISSION OF ELECTRONS FROM HOT BODIES.[1]

BY H. L. COOKE AND O. W. RICHARDSON.

THE following experiments to detect and measure this effect have been made with filaments of osmium which were kindly supplied to us by the Deutsche Gasglühlicht Actiengesellschaft, of Berlin. The filament was placed in one arm of a sensitive Wheatstone's bridge which was actuated by the main heating current C. Two of the arms carried high resistances so that 99 per cent. of the current C flowed through the filament and the adjacent arm which carried a standard resistance A of 10 ohms. The filament was surrounded by a brass cylinder from which it was insulated. The cylinder was connected in series with a microampere-meter, suitably shunted, and a suitable source of electromotive force V, which could be reversed, to a point midway between two equal resistances. The two ends of these resistances were connected either (1) to the opposite ends of the filament (filament shunted) or (2) to the opposite ends of the standard resistance A (standard shunted).

The resistances were first adjusted so that there was no deflection of the galvanometer when the electromotive force V prevented the thermionic current escaping from the filament. The deflection of the galvanometer D_T and the magnitude T of the thermionic current were then observed when the potential difference V was reversed. This deflection is due to the following causes: (1) the change in the resistance of the filament which is due to the cooling effect under investigation, (2) the change in the Joule heating effect due to the superposition of the thermionic current in the wire on the original heating

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

current and (3) a disturbance of the balance of the bridge circuit which arises from the thermionic current acting like a battery. The two last effects are different according as the filament or the standard is shunted. It can be shown that, to the first order of small quantities, the deflection which is due to these effects does not reverse when the main current is reversed, whereas the part which is due to (1) does reverse under these conditions. In this way these disturbing effects may be eliminated.

To reduce the deflections to absolute values a known change in the Joule heating effect was produced by changing the total resistance R of the bridge circuit by a known amount. This was done by placing a shunt of 100 ohms across a standard 1 ohm resistance in series in R. If the galvanometer deflection thus caused is D_E it is easy to show that with the numerical values of the resistances stated above

$$\varphi = 9.9 \times 10^{-3}\left[\left(\frac{C^3R_fD_T}{ETD_E}\right)_1 + \left(\frac{C^3R_fD_T}{ETD}\right)_2\right],$$

where φ is the cooling effect per unit thermionic current expressed in equivalent volts, E is the electromotive force in the heating circuit and the suffixes 1 and 2 denote that the readings are taken with the main heating current in opposite directions in the two cases. The above formula is approximate and subject to certain corrections which it would take too much space to discuss here.

All the quantities C, V and R_f have been varied by factors of about 2 and D_E and D_T by much larger factors without changing φ by more than the error to be expected from the method of measurement. Experiments have been made with values of T varying from 5×10^{-6} amp. to 869×10^{-6} amp. The variation of T was produced by varying the temperature of the filament. There is no certain evidence that φ varies with the temperature, with the magnitude of the thermionic current or with the age of the filament; although the values of φ for the largest values of T are about 2 per cent. above the average and it is possible that this represents a real variation of φ with the temperature and is not due to experimental error.

Altogether 38 measurements of φ have been made. The mean of these gives $\varphi = 4.7$ volts. With the exception of three values, which were subject to large possible errors as they involved measurements of very small deflections, none of the remaining 35 values differ by more than 8 per cent. from the mean and most of them are very near it. This value of φ is in good agreement with these which have been obtained by two entirely independent methods: (1) from the heating effect when electrons are absorbed by metals and (2) from the variation with temperature of the emission of electrons from hot metals. The experiments therefore appear conclusively to demonstrate the existence of the effect under investigation.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

ON THE CAUSE OF THE APPARENT DIFFERENCES BETWEEN SPARK AND ARC SOURCES IN THE IMPARTING OF INITIAL SPEEDS TO PHOTO-ELECTRONS.[1]

BY R. A. MILLIKAN.

SOME months ago I reported in some detail[2] experiments which seemed to show that light of a given wave-length from a spark source possesses quite different photo-electric properties from light of the same wave-length from a quartz-mercury lamp.

In endeavoring to find a satisfactory explanation of these differences I made the following statement,[2] "The simplest assumption would be that these effects (of spark sources) are not purely photo-electric, but that the high initial velocities are produced by the action of long electromagnetic waves upon electrons already liberated by the light. It is to be observed that all the spark curves look like the simple superposition upon the ordinary photo-curve of some influence which throws part of the already freed electrons (*e. g.*, those ejected toward one side) away from the surface, thus producing the high velocity feature of the curve, while it drives the other part back against the surface, thus making necessary the application of strong accelerating potentials to overcome this backward push. Nevertheless, the (two) experiments previously mentioned seem to speak decisively against this assumption and to show that the inward and outward forces on the electrons must be looked for in some influence of the light itself." Of the two experiments referred to the one which was considered crucial was as follows. The spark and mercury arc sources were placed so that the light from both could fall simultaneously upon the electrode. Then, while the spark was kept running the light from it was prevented from falling on the electrode, while that from the mercury arc was allowed so to fall. This procedure, it was thought, would give opportunity for the electromagnetic disturbances to impart the high velocities characteristic of spark sources to the electrons liberated by the mercury arc. But instead of this being the case, *the velocities were always exactly those characteristic of the mercury source.* Just as soon, however, as the light from the spark, in addition to that from the mercury arc, was allowed to fall upon the electrode the high velocities characteristic of the spark source were obtained. This seemed to show conclusively that these high velocities were due to the light itself and not to secondary effects of any kind, since these secondary effects were all operating when the light itself was screened off. The second experiment which supported strongly this conclusion consisted in enclosing the whole spark-producing apparatus in an iron box *A*, Fig. 1, the walls of which were 2 mm. thick, without finding that the character of the curves obtained was materially altered. In view of the apparently conclusive evidence of these experiments I sought the explanation of the peculiar effect

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Verb. d. D. Phys. Ges., 14, page 712, 1912.

of the spark source in the extraordinary instantaneous intensity of spark sources and in the assumption of a dependence of initial velocity upon intensity when the latter was very great, but I promised further tests of this assumption as soon as opportunity could be found.

Upon returning to Chicago two months ago I asked Mr. W. H. Kadesch to take the tube shown in Fig. 2, which was still set up as it had been in my earlier experiments and to first duplicate my curves with the arrangement of Fig. 1, the condensers *C* being close to the spark gap (very small self induction was found, in the earlier experiments, an essential condition of high velocities):

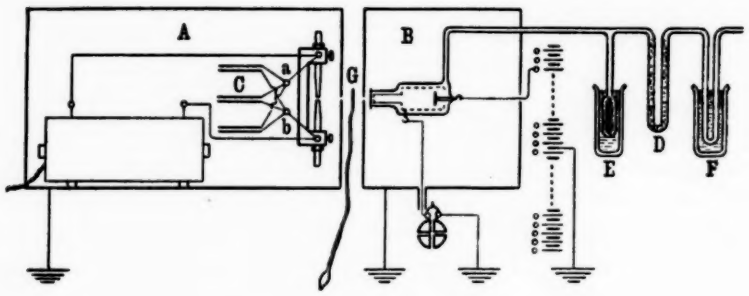

Fig. 1.

then to make a heavily insulated non-inductive spark circuit 30 ft. long and remove all the spark apparatus save the gap to another room. The spark circuit then led from the jars through the straight non-inductive circuit to the spark gap, all portions of this circuit being in the plane of the figure (Fig. 1). When this was done the positive potential fell from 60 volts to 10 volts and the photocurrent-potential curve was almost identical with that furnished by the mercury lamp. When, however, the spark gap was pulled six feet away from the photo-cell the positive potential slowly fell to 1.6 volts and the curves from the two sources became identical. When next the spark gap was kept in its position and the spark-producing apparatus was moved up toward the photo-cell the positive potential began to rise and ultimately reached over 100 volts. Shifting the position of the coil would change this markedly.

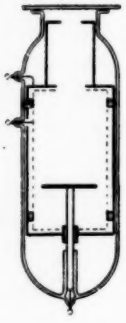

Fig. 2.

These experiments seem to furnish altogether conclusive evidence that the first assumption reported in the preceding paper was correct and that, despite the evidence to the contrary, the peculiarities of the photo-curves from spark sources are all due to disturbances having their origin in the spark circuit.

It becomes necessary, however, to explain the two experiments which led to the opposite conclusion. With regard to the second, viz., the lack of effect of all ordinary as well as extraordinary screening devices (both *A* and *B*, Fig. 1, were completely closed boxes of heavy sheet iron) in cutting down these dis-

turbances, there is nothing to be said except that no effective screening from such disturbances seems to be possible with spark sources, at least under the circumstances here used, in which the coil had to be actuated by an alternating current led into its primary from an outside circuit. This may have important bearings on several other types of experiments.

With regard to the first, we must assume that the time during which the disturbances sent out from the continuously running spark circuit can act on the electrons liberated by the mercury light is less than 1/1,000 of the time interval between these impulses since otherwise the running of the spark circuit, when its light was screened off from the photo-cell, would have caused the positive potential to rise observably above that due to the mercury light.

These results are reported at once even though all the obscurities are not yet removed, in order that others who may have been interested in the preceding report may know that I am now convinced that the anomalous results obtained with spark sources must be explained by a consideration of electromagnetic disturbances rather than by the assumption of a dependence of energy of emission upon intensity. *In fact the final practical identity of the spark and mercury-arc potentials (the two sources were of about the same mean intensity) shows that the intensity may vary at least in the ratio 1,000 to 1 without influencing appreciably the velocity of emission of photo-electrons.*

Note added January 15, in correcting proof. — I have learned since the above paper was prepared that Pohl and Pringsheim have recently reached similar conclusions (Verh. d. D. Phys. Ges., Nov. 15, 1912), and have supported them by other types of evidence than that herewith presented.

THE STABILITY OF RESIDUAL MAGNETISM.[1]

BY N. H. WILLIAMS.

A SIXTY cycle unidirectional current varying between zero and a maximum value is made to produce a demagnetizing influence upon a magnetized specimen of iron or steel. Demagnetization and recovery curves are plotted by an oscillographic method for different values of residual magnetism.

In a given specimen, for any value of residual magnetism, there is a definite elastic limit, which is the maximum field that can be applied without permanent demagnetization. Within this limit, magnetic stress and strain are approximately proportional.

The hysteresis for this kind of cycle is very small and is not greater for very hard steel than for soft iron.

The extreme points of the demagnetization curves lie upon a smooth curve which lies inside the maximum hysteresis curve and parallel to it.

The susceptibility to small demagnetizing fields increase slightly with decrease of residual magnetism, the maximum difference observed being about 8 per cent.

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

The recovery from the effect of a demagnetizing field increases with decrease of remanence and for hard steel reaches a value as high as 24 per cent. of the maximum residual induction. For softer specimens, the recovery is less.

There is shown to be a well defined upper limit beyond which the residual magnetism of an open magnetic circuit cannot be made to pass.

SOME DIRECT MEASUREMENTS OF THE TEMPERATURE CORRECTION TO THE REFRACTIVE INDEX OF WATER AROUND 20° C.[1]

BY F. A. MOLBY.

THE interference apparatus, previously described before the Society, was used for making direct measurements of the change in the *optical length* of a column of water 1.3 cm. in actual length. A fused quartz ring, cut away to allow water circulation, was used for an annulus separating two partially silvered glass plates, between which the column of water was used as an optical medium instead of air or vacuum used by Scheel, Dorsey, and others when they were measuring the expansion of materials at various temperatures. A mercury thermostat was used to control the temperature at any desired value with a high degree of accuracy, and the temperature changes in the water were measured with five thermocouples connected in series.

Several different sets of measurements were made and second degree equations were obtained for the relation between temperature and refractive index correction. It is believed that for the wave-length of light used the correction to the refractive index has been determined to a degree of accuracy such that the errors are not greater than 0.5 per cent. of the whole correction, for ranges of several degrees above and below 20° C.

THE EFFECTS OF GAS CURRENTS ON SOUND, WITH APPLICATIONS TO VENTILATING SYSTEMS OF AUDITORIUMS.[1]

BY F. R. WATSON.

A NUMBER of recorded experiments indicate that gas currents have an effect on the progress of sound waves. Lord Rayleigh has shown[2] that reflection at the boundary of gaseous media depends on the difference in the velocities of sound in the two media. Taking $V = \sqrt{\gamma p/\rho}$, an increase in temperature or moisture decreases ρ and hence increases V. With constant temperature, changes in p do not affect V, since p/ρ remains constant. Since γ is not the same for all gases, the value for V is thus further likely to change. Total reflection may occur for oblique incidence. Refraction also occurs when the velocities of sound are different in the two media. Furthermore, air in motion carries sound with it. Experiments have been carried out that verify some of these conclusions. For instance, the sound from a Galton's whistle

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Theory of Sound, Vol. II., section 270.

has been reflected from the sheet of hot gases rising above a straight row of gas burners. The receiver was a Barry sensitive flame. Reflection has also been obtained from the boundary between carbon dioxide and air. Another set of experiments has shown that hot currents of gas in a room absorb but little sound energy. It has also been shown that irregular air currents in a room have a pronounced action of the sound. This effect follows when a Rayleigh suspended disc resonator responds to the sound of an organ pipe across the room. With no air currents, the deflection is steady; with air currents, the deflection is erratic. The theory indicates that total transmission may take place for a certain angle of incidence. This conclusion is contrary to the case for other forms of radiation, and attempts are being made to test the statement. The application of these results to the case of a ventilating system in an auditorium shows that little effect is to be expected for ordinary conditions unless the air currents are very irregular. For extreme variation in temperature and other factors, an effect may be found. Examples may be cited to show these different effects of ventilating systems.

Air Resistance to Falling Bodies.[1]

By A. A. Somerville.

EXPERIMENTS have been performed to determine the time required for small spheres of various sizes to fall through a considerable distance. The falling body has been started by closing circuit through an electro-magnet thereby opening a trap-door. The time of fall is recorded on a chronograph,

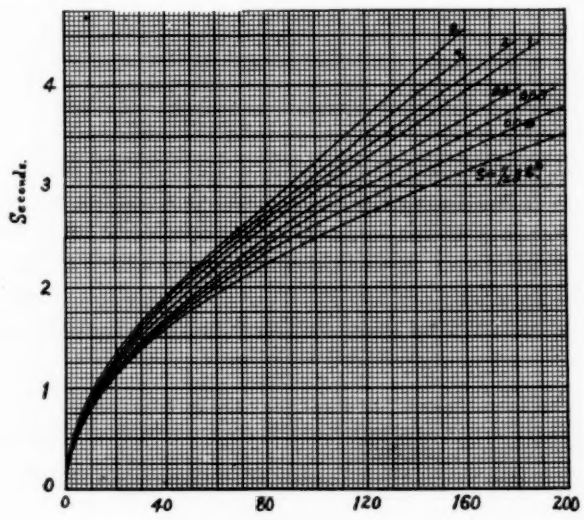

Fig. 1.

the stylus being connected with the same circuit as the magnet which releases the ball and also with another circuit which is closed when the ball strikes at

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

the bottom or end of the distance fallen. A net as large as a pocket handerchief will generally catch the ball after a fall of 200 feet through quiet air. The experiments have been tried in three places: an elevator shaft, 70 feet; a street car bridge over a gorge 120 feet deep; a cable over a gorge 360 feet deep. Seven sizes of shot have been used, also marbles, steel balls and loaded phials. Curves are given, showing the time required for shot of various sizes to fall given distances.

DISTRIBUTION OF THE ELEMENTS IN THE SOLAR ATMOSPHERE.[1]

BY CHARLES E. ST. JOHN.

THE displacement of the Fraunhofer lines in the outer edges of the penumbra of sun-spots was first observed by Evershed, who attributed the displacements to the outward motion of the material. An investigation covering some five hundred lines has confirmed Evershed's observation and explanation. When one considers lines of intensity 0 to 10, the displacements for lines of the same element *increase* with *decrease* of the intensities and differ for lines of like intensity of different elements, and when the displacements are reduced to a common wave-length they vary as the wave-length. For the lines of intensities 1 and 2 of iron the result of the comparison between lines in the violet, green, and red is as follows:

	Absolute Displacements.		Reduced to λ 5000.	
	1.	2.	1.	2.
Violet	0.0220 Å	0.0200 Å	0.0262	0.0240
Green	0.0299	0.0259	0.0296	0.0259
Red	0.0318	0.0308	0.0268	0.0250

This relation between displacement and wave-length is the hall-mark of the Doppler effect and seems decisively in favor of motion as the cause of the phenomena. For lines of the intensity under consideration the displacements are such as to indicate an outward flow increasing with the depth, on the assumption that the weak lines are on the average at a low level. Assembling all the data for the 192 iron lines, we have the following scale connecting intensities and displacements reduced to λ5000.

Intensities	0	1	2	3	4	5	6	7	8	10
Displacements	0.0285 Å	0.0278	0.0249	0.0229	0.0209	0.0193	0.0165	0.0134	0.0095	0.0050

With this scale established it is possible to determine the level of the lines of other elements in terms of the iron scale.

For lines of intensity exceeding 10 on Rowland's scale the displacements for all lines become exceedingly small, change sign, and then increase in absolute

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

value with the intensities of the lines reaching the highest negative value for the H_3 and K_3 lines of calcium. The negative sign indicates inward motion of the upper levels of the solar atmosphere.

When the 27 substances are arranged in a series beginning at the left with the element appearing at the highest level, it is seen at a glance that high atomic weights occur more frequently toward the right, pointing to a low level even for the strongest lines of the heavier elements. This becomes more evident when taken in five groups from left to right, with seven elements in the first group, and the atomic weights summed for each group and the mean atomic weight of the group found.

Groups.	1	2	3	4	5
Sum of atomic weights.....	258	242	317	526	727
Mean atomic weights.....	37	48	63	105	145

ON THE VALUE OF e.[1]

BY R. A. MILLIKAN.

THIS paper contains a summary of the results obtained in the work of the past two years in the determination of all of the factors entering into the oil drop method for the evaluation of e.

These results are as follows:

If μ_{23} is the coefficient of viscosity of dry air at 23° C. then

$$\mu_{23} = 1824 \pm .00015 \times 10^{-7}. \quad (1)$$

Or if μ_t is the coefficient of viscosity at any temperature between 12° C. and 28° C. then

$$\mu_t = .0001824\{1 + .0027(t - 23)\}. \quad (2)$$

If the mean free path of a molecule of air is computed from $\mu = .350\rho\bar{c}l$ where $\bar{c}$ is the *average* molecular velocity, then, in the formula

$$x = 6\pi\mu a v_1 \left(1 + A\frac{l}{a}\right)^{-1}, \quad (3)$$

x being the force necessary to impart to a spherical drop of radius a velocity v_1 then

$$A = .874.[2]$$

But when $l/a > .4$ equation (3) must be replaced by

$$x = 6\pi\mu a v_1 \left\{1 + \frac{l}{a}\left(.874 + .32\epsilon^{-1.24\frac{l}{a}}\right)\right\}^{-1}.$$

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Owing to a failure to see proof, incorrect numbers for these constants got into a note in the PHYSICAL REVIEW, Vol. XXXIV., p. 398. These former numbers were never obtained in any experiments, but represented simply typographical errors.

If e denotes the value of the elementary electrical charge then the present investigation gives

$$e = 4.774^2 \pm .008 \times 10^{-10} \text{ E.S. units.}$$

A SUBSTITUTE FOR A BRONSON RESISTANCE.[1]

BY DAVID W. CORNELIUS AND J. G. KEMP.

IN the Philosophical Magazine for August, 1911, pp. 301, 302, Campbell describes certain liquid resistances which may be used for the same purposes as that generally known as the Bronson resistance.

The authors, in October, 1911, while working on photo-electric problems, found it very convenient to use a high resistance in order to measure electrical currents of the order of magnitude 10^{-11}, 10^{-13} amp. From the suggestions obtained from the article by Campbell liquid resistances were made using C.P. benzol and absolute alcohol, and C.P. meta-xylol and absolute alcohol. The mixtures were placed in straight glass tubes 1.7 cm. in diameter, about 15 cm. in length, with one electrode sealed in the bottom and the other electrode passed through an insulating stopper in the top. The upper electrode was made movable in order that the distance between the electrodes could be changed.

The material of the stoppers were in some cases hard rubber and in others amber. Owing to the action of the vapor of the mixture upon the hard rubber and the amber it was found impractical to use this type of insulating stopper for any other purpose than that of investigating the effect of change of distance between the electrodes.

In measurements for constant distance between the electrodes straight glass tubes of 1 cm. diameter, about 10 cm. in length were used with electrodes sealed in the lower end and in the side about 2 cm. and 3 cm. apart, and having a ground glass stopper in the upper end. This type of cell avoided the escape of any of the contents of the tube by vaporization, thus keeping the mixture constant.

Ohm's law was found to hold for voltage less than one volt and in some cases as high as 720 volts.

Tables of data are given below which show first the relation between the resistance and the distance between the electrodes for different percentages of alcohol; second, the relation between the resistance and the per cent. of alcohol in the mixture for benzol and meta-xylol.

The observations show that the relation between resistance and electrode distance is of the form $R = R_0 + \alpha x$ provided the distance between the electrodes does not exceed 3 cm.

R_0 varies from 6×10^{10} ohms for the mixture of benzol with 6.6 per cent. alcohol to about 5×10^8 ohms for a similar mixture containing 17 per cent. alcohol.

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

TABLE I.

C_6H_6 + 6.6 per cent. C_2H_5OH.	$C_6H_4(CH_3)_2$ + 5.6 per cent. C_2H_5OH.
Max. voltage used 406 *volts.*	*Max. voltage used* 720 *volts.*
Min. voltage used 81 *volts.*	*Min. voltage used* 640 *volts.*
Diam. of tube 1.8 *cm.*	*Diam. of tube* 0.74 *cm.*
Temp. between 23°–24° *C.*	*Temp. between* 23°–24° *C.*
Amber stopper.	*Hard-rubber stopper.*

Resistance in Ohms.	Electrode Distance, Cm.	Resistance in Ohms.	Electrode Distance, Cm.
22.2×10^{10}	3.695	99.0×10^{10}	5.035
15.0×10^{10}	2.575	66.0×10^{10}	4.105
13.0×10^{10}	1.610	49.5×10^{10}	3.025
11.6×10^{10}	1.085	36.5×10^{10}	2.140
10.7×10^{10}	0.725	23.9×10^{10}	1.125
9.3×10^{10}	0.375	15.0×10^{10}	0.540
8.6×10^{10}	0.090	11.6×10^{10}	0.275

TABLE II.

C_6H_6 + 10.3 per cent. C_2H_5OH.	$C_6H_4(CH_3)_2$ + 10.1 per cent. C_2H_5OH.
Max. voltage used 406 *volts.*	*Max. voltage used* 640 *volts.*
Min. voltage used 6 *volts.*	*Min. voltage used* 200 *volts.*
Diam. of tube 1.60 *cm.*	*Diam. of tube used* 0.735 *cm.*
Temp. between 24°–25° *C.*	*Temp. between* 24°–25° *C.*
Amber stopper.	*Hard-rubber stopper.*

Resistance in Ohms.	Electrode Distance, Cm.	Resistance in Ohms.	Electrode Distance, Cm.
9.5×10^{9}	3.500	19.9×10^{9}	6.010
7.2×10^{9}	2.850	16.3×10^{9}	4.935
6.3×10^{9}	2.110	12.7×10^{9}	3.765
5.9×10^{9}	1.800	10.0×10^{9}	2.775
5.5×10^{9}	1.435	7.2×10^{9}	1.715
5.3×10^{9}	1.190	4.3×10^{9}	0.649
4.8×10^{9}	0.715	3.5×10^{9}	0.395
4.0×10^{9}	0.285	3.1×10^{9}	0.240

It was found that the resistances remained fairly constant for a period of about ten days. When evaporation was permitted to take place the resistance increased. This was due to the escape of alcohol.

The variation with the temperature was not investigated except that no change was noticeable for changes between 24° C. and 25° C. Since this work was completed, however, Campbell, in a note in the Philosophical Magazine for April, 1912, p. 668, shows that the resistance made of commercial xylol and absolute alcohol had a temperature coefficient. Between 15° C. and 30° C. the resistance at a temperature t is given by

$$R_t = R_{20}(1 + 0.014(t - 20)).$$

The method used for measuring the resistance was a simple circuit with the

TABLE III.

C_6H_6 + 17 per cent. C_2H_5OH. *Max. voltage used* 41 *volts.* *Min. voltage used* 6 *volts.* *Diam. of tube* 1.68 *cm.* *Temp. between* 24°–25° *C.* *Hard-rubber stopper.*		$C_6H_4(CH_3)_2$ + 16.3 per cent. C_2H_5OH. *Max. voltage used* 120 *volts.* *Min. voltage used* 6 *volts.* *Diam. of tube* 0.735 *cm.* *Temp. between* 24°–25° *C.* *Hard-rubber stopper.*	
Resistance in Ohms.	**Electrode Distance, Cm.**	**Resistance in Ohms.**	**Electrode Distance, Cm.**
11.4×10^8	5.700	15.5×10^8	5.420
8.2×10^8	4.575	11.4×10^8	4.360
7.2×10^8	3.200	9.5×10^8	3.625
6.5×10^8	2.125	7.3×10^8	2.620
5.9×10^8	1.330	5.6×10^8	1.716
5.6×10^8	0.910	3.9×10^8	0.955
5.3×10^8	0.580	2.9×10^8	0.430
4.8×10^8	0.230	2.7×10^8	0.220

TABLE IV.

C_6H_6 + C_2H_5OH. *Max. voltage used* 406 *volts.* *Min. voltage used* 81 *volts.* *Diam. of tube* 1.60 *cm.* *Temp. between* 24°–25° *C.* *Electrode distance* 1.435 *cm.* *Amber stopper.*		$C_6H_4(CH_3)_2$ + C_2H_5OH. *Max. voltage used* 720 *volts.* *Min. voltage used* 560 *volts.* *Diam. of tube* 0.735 *cm.* *Temp. between* 24°–25° *C.* *Electrode distance* 1.435 *cm.* *Hard-rubber stopper.*	
Resistance in Ohms.	**Per Cent. Alcohol.**	**Resistance in Ohms.**	**Per Cent. Alcohol.**
8.9×10^{10}	6.6	18.6×10^{10}	5.7
2.6×10^{10}	8.0	6.9×10^{10}	7.2
1.1×10^{10}	9.2	2.2×10^{10}	8.8
0.55×10^{10}	10.3	1.5×10^{10}	9.6
0.06×10^{10}	17.0	1.2×10^{10}	10.5
		0.05×10^{10}	16.3

resistance, storage cells, and a d'Arsonval galvanometer in series. The sensibility of the galvanometer was 3.7×10^{-10} ampere per mm. deflection.

A check on this method was made by using the leakage of a charged condenser through the resistance and measuring the rate of leak by means of an electrometer. It was found that the two methods gave values for the resistances which agree very closely.

The glass tubes were mounted on hard-rubber insulating stands to prevent leakage.

In addition to using these resistances in place of the Bronson resistance they may be used for teaching purposes in laboratory work in determining the figure of merit of a galvanometer, the comparison method of measuring insu-

lation resistance of cables, and the experiment on measuring a high resistance by the leakage of a charged condenser.

THE RELATION BETWEEN CURRENT AND TIME RATE OF CHANGE OF ELECTRIC FORCE.[1]

BY MAX MASON.

A CONSISTENT atomic theory of electricity should imply that the electron acts as a unit, and terms should not be present in the fundamental equations which describe the action of parts of the unit. The introduction of the concept of density of charge as the number of electrons per unit volume times the electronic charge, is natural and advisable in the study of material bodies. It would be difficult to define density consistently with an atomic theory of electricity so that the concept could be applied to the electron itself. If the electron be assumed as a "point charge" a simplification of electrodynamic theory results, since it may then be shown that electric current appears as a manifestation of time rate of change of electric force. The equation

$$\text{curl } \mathbf{h} = \dot{\mathbf{e}} \tag{1}$$

is here assumed for the electron theory, in place of the usual form

$$\text{curl } \mathbf{h} = \dot{\mathbf{e}} + \rho\mathbf{v}. \tag{2}$$

By the application of statistical methods an equation for material bodies is derived from (1). If $\mathbf{E}$ and $\mathbf{H}$ be written for the mean values of $\mathbf{e}$ and $\mathbf{h}$ then the equation

$$\text{curl } \mathbf{H} = \dot{\mathbf{E}} + \rho\mathbf{v}$$

is obtained, the term $\rho\mathbf{v}$ for current, and the term $\dot{\mathbf{E}}$ both appearing as a consequence of the mean value of $\dot{\mathbf{e}}$. Thus the magnetic effect of electrons in motion is entirely accounted for by the time rate of change of electric force, and the term $\rho\mathbf{v}$ in equation (2) is redundant. The analytical reason for this is the fact that the time mean of the derivative of a discontinuous function differs from the derivative of the time mean of the function by a term which expresses the gain per second in the function due to its discontinuities.

THE EVIDENCE THAT SODIUM BELONGS TO A RADIOACTIVE SERIES OF ELEMENTS.[1]

BY F. C. BROWN.

CAMPBELL and Wood[2] could not detect in the compounds of sodium any activity that was definitely greater than that common to all matter and certainly no activity one thousandth as great as that of potassium. If therefore sodium belongs to a radioactive series of elements, it of itself must

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Proc. Camb. Phil. Soc., 14, p. 15.

be undergoing disintegration and at the same time be inactive so far as measurable ionizing radiations are concerned, or sodium must be a relatively inactive product resulting from a radioactive parent. If the former is true then sodium left at rest a long period of time should diminish in amount. But if the latter presumption is true, sodium should in time form from one or more elements in appreciable quantities.

For evidence as to these presumptions the facts of geo-chemistry are used and it is found that two separate sets of facts favor the hypothesis that sodium has been accumulating radioactively over the land during geologic history. First an investigation of the sodium carried to the ocean by the rivers annually, and also an investigation of the total sodium content of the ocean, seems to show that the age of the ocean and of the earth is probably not more than 75,000,000 years old. Joly[3] and also F. W. Clarke[4] after a careful consideration of the possible sources of error deduce these figures. On the other hand if the age is based on the amounts of lead and helium associated with uranium in minerals, we find that the age is upwards of a billion years. Thus far the fact seems to be that there is not as much sodium in the ocean as expected. The apparently sufficient explanation is that sodium has been accumulating radioactively over the land but not so over the ocean. Thus the rivers formerly should not have carried as much sodium as they now carry.

An investigation of a second set of facts also makes it convenient to put forth the hypothesis of the radioactive accumulation of sodium over the land areas. If we compare the annual additions of sodium and chlorine by the rivers, it is found that more sodium than chlorine is carried to the ocean. While an examination of the contents of the ocean water reveals much more chlorine than sodium, these two elements are compared because both are soluble in all compounds and neither is deposited in appreciable quantities in the ocean sediments. Of course this comparison only indicates that sodium has accumulated more rapidly than chlorine. So far as the argument is concerned chlorine may also have accumulated radioactively.

No other common explanation has been offered for the two distinct sets of comparative data here used. It would seem therefore worth while to make a search for the possible parentage of sodium. The parent or parents should commonly exist over the land areas, and should in all compounds be relatively insoluble in water. If however the parent has become almost extinct this search would be futile.

[3] Phil. Mag., 6, 22, p. 357, 1911.

[4] The Data of Geo-Chemistry, 2d edition, Bulletin 491, N. S. G. S.